

Finance II (Dirección Financiera II)

Apuntes del Material Docente

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FIXED-INCOME SECURITIES

Bond characteristics

- A bond is a security that is issued in connection with a borrowing arrangement.
- The borrower issues (i.e. sells) a bond to the lender for some amount of cash.
- The arrangement obliges the issuer to make specified payments to the bondholder on specified dates.

Bond characteristics

- A typical bond obliges the issuer to make semiannual payments of interest to the bondholder for the life of the bond.
- These are called **coupon payments**.
- Most bonds have coupons that investors would clip off and present to claim the interest payment.

Bond characteristics

- When the bond matures, the issuer repays the debt by paying the bondholder the bond's **par value** (or **face value**).
- The coupon rate of the bond serves to determine the interest payment:
- The annual payment is the coupon rate times the bond's par value.

Bond characteristics

- The contract between the issuer and the bondholder contains:
1. **Coupon rate**
 2. **Maturity date**
 3. **Par value**

Example

- A bond with par value EUR1000 and coupon rate of 8%.
- The bondholder is then entitled to a payment of 8% of EUR1000, or EUR80 per year, for the stated life of the bond, 30 years.
- The EUR80 payment typically comes in two semiannual installments of EUR40 each.
- At the end of the 30-year life of the bond, the issuer also pays the EUR1000 value to the bondholder.

Zero-coupon bonds

- These are bonds with no coupon payments.
- Investors receive par value at the maturity date but receive no interest payments until then.
- The bond has a coupon rate of zero percent.
- These bonds are issued at prices considerably below par value, and the investor's return comes solely from the difference between issue price and the payment of par value at maturity.

Treasury bonds, notes and bills

- The U.S. government finances its public budget by issuing public fixed-income securities.
- The maturity of the **treasury bond** is from 10 to 30 years.
- The maturity of the **treasury note** is from 1 to 10 years.
- The maturity of the **treasury bill** (T-bill) is up to 1 year.

Corporate bonds

Like the governments, corporations borrow money by issuing bonds.

- Although some corporate bonds are traded at organized markets, most bonds are traded over-the-counter in a computer network of bond dealers.
- As a general rule, safer bonds with higher ratings promise lower yields to maturity than more risky bonds.

Corporate bonds

- There are several types of corporate bonds related to the specific characteristics of the bond contract:
 1. Call provisions on corporate bonds
 2. Puttable bonds
 3. Convertible bonds
 4. Floating-rate bonds

1. Call provisions on corporate bonds

- Some corporate bonds are issued with call provisions allowing the issuer to repurchase the bond at a specified **call price** before the maturity date.
- For example, if a company issues a bond with a high coupon rate when market interest rates are high, and interest rates later fall, the firm might like to retire the high-coupon debt and issue new bonds at a lower coupon rate. This is called **refunding**.

1. Call provisions on corporate bonds

- Callable bonds are typically come with a period of call protection, an initial time during which the bonds are not callable.
- Such bonds are referred to as **deferred callable bonds**.

2. Puttable bonds

- While the callable bond gives the issuer the option to retire the bond at the call date, the **put bond** gives this option to the bondholder.

3. Convertible bonds

- Convertible bonds give bondholders an option to exchange each bond for a specified number of shares of common stock of the firm.
- The **conversion ratio** is the number of shares for which each bond may be exchanged.

3. Convertible bonds

- The **market conversion value** is the current value of the shares for which the bonds may be exchanged.
- The **conversion premium** is the excess of the bond value over its conversion value.
- **Example:** If the bond were selling currently for EUR950 and its conversion value is EUR800, its premium would be EUR150.

4. Floating-rate bonds

- These bonds make interest payments that are tied to some measure of current market rates.
- For example, the rate can be adjusted annually to the current T-bill rate plus 2%.

Other specific bonds

- Other bonds (or bond like assets) traded on the market:
 1. **Preferred stock**
 2. **International bond**
 3. **Bond innovations**

1. Preferred stock

- Although the preferred stock strictly speaking is considered to be equity, it can be also considered as a bond.
- The reason is that a preferred stock promises to pay a specified stream of dividends.
- Preferred stocks commonly pay a fixed dividend.
- Therefore, it is in effect a perpetuity.

1. Preferred stock

- In the last two decades, floating-rate preferred stocks have become popular.
- Floating-rate preferred stock is much like a floating-rate bond:
- The dividend rate is linked to a measure of current market interest rates and is adjusted at regular intervals.

2. International bonds

- International bonds are commonly divided into two categories:
2a. Foreign bonds
2b. Eurobonds

2a. International bonds Foreign bonds

- **Foreign bonds** are issued by a borrower from a country other than the one in which the bond is sold.
- The bond is denominated in the currency of the country in which it is marketed.
- For example, a German firm sells a dollar-denominated bond in the U.S., the bond is considered as a foreign bond.

2a. International bonds Foreign bonds

- **Foreign bonds** are given colorful names based on the countries in which they are marketed:
 1. Foreign bonds sold in the U.S. are called **Yankee bonds**.
 2. Foreign bonds sold in Japan are called **Samurai bonds**.
 3. Foreign bonds sold in U.K. are called **bulldog bonds**.

2b. International bonds Eurobonds

- **Eurobonds** are bonds issued in the currency of one country but sold in other national markets.
 1. The **Eurodollar** market refers to dollar-denominated Eurobonds sold outside the U.S..
 2. The **Euroyen** bonds are yen-denominated Eurobonds sold outside Japan.
 3. The **Eurosterling** bonds are pound-denominated Eurobonds sold outside the U.K..

Innovations in the bond market

- Issuers constantly develop innovative bonds with unusual features. Some of these bonds are:
 1. **Inverse floaters**
 2. **Asset-backed bonds**
 3. **Catastrophe bonds**
 4. **Indexed bonds**

1. Inverse floaters

- These are similar to the floating-rate bonds except that the coupon rate on these bonds **falls** when the general level of interest rates **rises**.
- Investors suffer doubly when rates rise:
 1. The present value of the future bond payments decreases.
 2. The level of the future bond payments decreases.
- Investors benefit doubly when rates fall.

2. Asset-backed bonds

- In asset-backed securities, the income from a specific group of assets is used to service the debt.
- For example, **Dawid Bowie bonds** have been issued with payments that will be tied to the royalties on some of his albums.

2. Mortgage-backed bonds

- Another example of **asset-backed bonds** is **mortgage-backed security**, which is either an ownership claim in a pool of mortgages or an obligation that is secured by such a pool.
- These claims represent **securitization** of mortgage loans.
- Mortgage lenders originate loans and then sell packages of these loans in the **secondary market**.

2. Mortgage-backed bonds

- The mortgage originator continues to service the loan, collecting principal and interest payments, and **passes these payments** to the purchaser of the mortgage.
- For this reason, mortgage-backed securities are called **pass-throughs**.

3. Catastrophe bonds

- **Catastrophe bonds** are a way to transfer catastrophe risk from a firm to the market.
- For example, Tokyo Disneyland issued a bond with a final payment that depended on whether there has been an earthquake near the park.

4. Indexed bonds

- **Indexed bonds** make payments that are tied to a general price index or the price of a particular commodity.
- For example, Mexico issued 20-year bonds with payments that depend on the **price of oil**.
- The U.S. Treasury issued inflation protected bonds in 1997. (**Treasury Inflation Protected Securities, TIPS**)
- For TIPS, the coupon and final payment is related to the **consumer price index**.

BOND PRICING

Bond pricing

- Because a bond's coupon and principal repayments all occur in the future, the price an investor would be willing to pay for a claim to those payments depends on the value of dollars to be received in the future compared to dollars in hand today.
- This present value calculation depends on the market interest rates.

Bond pricing

- First, we simplify the present value calculation by assuming that there is **one** interest rate that is appropriate for discounting cash flows of any maturity.
- Later we can relax this assumption easily.
- In practice, there may be **different discount rates** for cash flows accruing in **different periods**.

Bond pricing

- To value a security, we discount its expected cash flows by the appropriate discount rate.
- The cash flows from a bond consist of coupon payments until the maturity date plus the final payment of par value.
- Therefore,
Bond value = Present value of coupons + Present value of par value

Bond pricing

- Let r denote the interest rate and T the maturity date of the bond. Then, the value of the bond is given by:

$$\text{Bond value} = \left[\sum_{t=1}^T \frac{\text{Coupon}}{(1+r)^t} \right] + \frac{\text{Par value}}{(1+r)^T}$$

Bond pricing

- Notice that the bond pays a T -year annuity of coupons and a single payment of par value at year T .
- Therefore, it is useful to introduce the **annuity factor (AF)** and the **discount factor (DF)**.

Annuity factor

- The ***T*-year annuity factor** is used to compute the present value of a *T*-year annuity:

$$T - \text{period annuity factor} = AF(r, T) = \frac{1}{r} \left[1 - \frac{1}{(1+r)^T} \right]$$

- Notice that AF is a function of the interest rate, *r* and the time period of the annuity, *T*.

Discount factor

- The **discount factor for period *t*** is used to compute the present value of a cash flow of year *t*:

$$\text{Discount factor of period } t = DF(r, t) = \frac{1}{(1+r)^t}$$

- Notice that DF is a function of the interest rate, *r* and the time of the cash flow payment, *t*.

Bond pricing

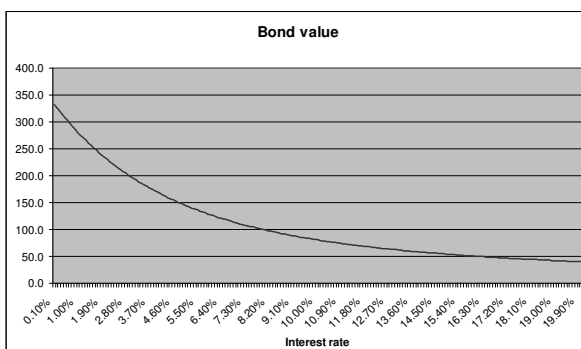
- We can rewrite to previous bond pricing formula using the AF and DF as follows:

$$\text{Bond value} = \text{Coupon} \times AF(r, T) + \text{Par value} \times DF(r, T)$$

Bond pricing

- The bond value has **inverse relationship** with the interest rate used to compute the present value.
- **Example:** We consider a 30-year bond with par value 100 and coupon rate 8%.
- In the following figure, we present the value of this bond as a function of the interest rate, *r*.

Bond pricing



Bond pricing: Perpetuities

- The value of a **perpetuity** paying *C* forever with yield *y* at time *t*=0 is

$$P = C/y$$

where the bond pays the following cash flows:

t=0	t=1	t=2	t=3	t=4	...
0	<i>C</i>	<i>C</i>	<i>C</i>	<i>C</i>	...

- We can prove this formula easily: Use that $a + ab + ab^2 + ab^3 + \dots = a/(1-b)$ when $|b| < 1$.

Bond pricing: Perpetuities

- The value of a **growing perpetuity** paying $C_{t+1}=(1+g)C_t$ with yield y and $g<y$ at time $t=0$ is

$$P = C_1/(y-g)$$

where the bond pays the following cash flows:

t=0	t=1	t=2	t=3	t=4
0	C_1	C_2	C_3	C_4

- We can prove this formula easily: Use that $a + ab + ab^2 + ab^3 + \dots = a/(1-b)$ when $|b|<1$.

Bond pricing: Annuity

- We prove that the price of the following annuity with yield y at time $t=0$

t=0	t=1	t=2	...	t=n	t=n+1	t=n+2...
0	C	C	...	C	0	0 ...

is

$$P = \frac{C}{y} \left[1 - \frac{1}{(1+y)^n} \right]$$

Bond pricing: Annuity

- In the proof, one uses the fact that the cash flow of the annuity is the difference of the following two perpetuities:

t=0	t=1	...	t=n	t=n+1	t=n+2	t=n+3	...
0	C	...	C	C	C	C	...
0	0	...	0	C	C	C	...

- Compute the price of both perpetuities and take the difference of the two prices to get the price of the annuity.

Bond pricing between coupon dates

Quoted bond prices

- In the newspapers, there are two prices presented for each bond:
- The **bid price** at which one can sell the bond to a dealer.
- The **asked price** is the price at which one can buy the bond from a dealer.
- The asked price is higher than the bid price.

Accrued interest and quoted bond prices

- The bond prices presented in the newspaper are not actually the prices that investors pay for the bond.
- The prices which appear in financial press are called **flat prices**.
- This is because the quoted price does not include the interest that accrues between coupon payment dates.

Accrued interest and quoted bond prices

- The actual invoice price that the buyer pays for the bond includes accrued interest:

$$\text{Invoice price} = \text{Flat price} + \text{Accrued interest}$$

Accrued interest and quoted bond prices

- If a bond is purchased between coupon payments, the buyer must pay the seller for accrued interest, the prorated share of the upcoming semiannual coupon.

Accrued interest and quoted bond prices

- **Example:** If 30 days have passed since the last coupon payment, and there are 182 days in the semiannual coupon period, the seller is entitled to a payment of accrued interest of 30/182 of the semiannual coupon.

Accrued interest and quoted bond prices

- In general, the formula for the amount of accrued interest between two dates of the semiannual payment is
Accrued interest =
(Annual coupon payment / 2) x
(Days since last coupon payment /
Days separating coupon payments)

Bond yields

Yield to maturity

- In practice, an investor considering the purchase of a bond is not quoted a promised rate of return.
- Instead, the investor must use the bond price, maturity date, and coupon payments to infer the return offered by the bond over its life.
- **The yield to maturity (YTM) is defined as the interest rate that makes the present value of a bond's payments equal to its price.**

Yield to maturity

- The calculate the YTM, we solve the bond price equation for the interest rate given the bond's price.
- The following YIELD(·) or RENDTO(·) function can be used in English and Spanish language Excel, respectively, to compute the yield to maturity:

YIELD(settlement, maturity, rate, pr, redemption, frequency [,basis])

- settlement:** The settlement date of the security.
- maturity:** The maturity date of the security.
- rate:** The annual coupon rate of the security.
- pr:** The price per \$100 face value.
- redemption:** The security's redemption value per \$100 face value.
- frequency:** The number of coupon payments per year:
 - 1 = annual
 - 2 = semi-annual
 - 4 = quarterly
- basis:** The type of day counting to use.
 - 0 = US 30/360
 - 1 = Actual/Actual
 - 2 = Actual/360
 - 3 = Actual/365
 - 4 = European 30/360

Yield to maturity

- YTM differs from the **current yield** of the bond, which is the bond's annual coupon payment divided by the bond price.
- For **premium bonds** (bonds selling above par value), coupon rate is greater than current yield, which is greater than yield to maturity:
YTM < Current yield < Coupon rate

Yield to maturity

- For **discount bonds** (bonds selling below par value), coupon rate is lower than current yield, which is lower than yield to maturity:
Coupon rate < Current yield < YTM

TERM STRUCTURE OF INTEREST RATES

TERM STRUCTURE OF INTEREST RATES

- The term structure of interest rates is represented by the **yield curve**.
- The **yield curve is a plot of yield to maturity (YTM) as a function of time to maturity**.
- If yields on different-maturity bonds are not equal, how should we value coupon bonds that make payments at many different times?

TERM STRUCTURE OF INTEREST RATES

- **Example:** Suppose that zero-coupon bonds with 1-year maturity sell at YTM $y_1=5\%$, 2-year zeros sell at YTM $y_2=6\%$ and 3-year zeros sell at YTM $y_3=7\%$.
- Which of these rates should we use to discount bond cash flows?
- ALL OF THEM.
- The trick is to **consider each bond cash flow** – either coupon or par value payment – **as a zero-coupon bond**.

TERM STRUCTURE OF INTEREST RATES

- **Example** (continued): Price a bond paying the following cash flow:

t=1 100
t=2 100
t=3 1100

Period	Cash flow	Rate	DF(r,t)	Present value
1	100	5%	0.952	95.2
2	100	6%	0.890	89.0
3	1100	7%	0.816	897.9
Bond value:				1082.2

TERM STRUCTURE OF INTEREST RATES

- **Example** (continued):
In the previous table, bond value is computed by the following formulas:

$$\text{Bond value} = \frac{100}{(1+y_1)} + \frac{100}{(1+y_2)^2} + \frac{1100}{(1+y_3)^3}$$

$$\text{Bond value} = \frac{100}{(1+5\%)} + \frac{100}{(1+6\%)^2} + \frac{1100}{(1+7\%)^3} = 1082.2$$

ZERO-COUPON BONDS

- In the previous example, we used the yields of the zero-coupon bonds to discount future cash flows.
- Zero-coupon bonds are **maybe the most important bonds** because they can be used to build up other bonds with more complicated cash flow structure.
- When the prices of zero-coupon bonds are known, they can be used to **price** more complicated bonds.

SPOT RATE

- Practitioners call the yield to maturity on zero-coupon bonds **spot rate** meaning the rate that prevails today for a time period corresponding to the zero's maturity.
- We denote the spot rates for the time periods $t=1,2,\dots,T$ as follows:
 $\{y_1, y_2, \dots, y_T\}$
- The sequence of the spot rates over $t=1, \dots, T$ defines the **SPOT YIELD CURVE**:
 $\{y_1, y_2, \dots, y_T\}$

SHORT RATE

- In contrast, the **short rate** for a given time interval (for example 1 year) refers to the interest rate for that interval available at different points of time of the future.
- We denote the **1-year short rate** for the time period t as:
 ${}_{t-1}r_t$
More generally, for time periods $1 \leq t \leq T$:
 $\{{}_0r_1, {}_1r_2, {}_2r_3, \dots, {}_{T-1}r_T\}$

SHORT RATE AND SPOT RATE

- We shall relate the spot and short rates in two alternative situations:
 1. Future interest rates are certain.
 2. Future interest rates are uncertain.

SHORT AND SPOT RATES: FUTURE RATES ARE CERTAIN

- The **spot rates** can be computed knowing the short rates because:

$$(1+y_t)^t = (1+{}_0r_1)(1+{}_1r_2)\dots(1+{}_{t-1}r_t)$$

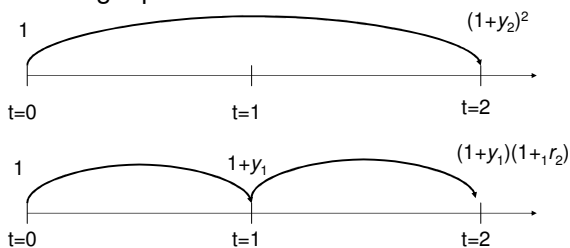
- Notice that $y_1 = {}_0r_1$.
- The **short rates** can be computed knowing the spot rates because:

$${}_{t-1}r_t = [(1+y_t)^t / (1+y_{t-1})^{t-1}] - 1$$

- THE ASSUMPTION IN THESE FORMULAS IS THAT FUTURE SHORT RATES ARE **KNOWN WITH CERTAINTY**.

SHORT AND SPOT RATES

- **Example:** Consider two alternative investment strategies for investing 1 euro during 2 periods:



SHORT AND SPOT RATES: FUTURE RATES ARE UNCERTAIN

- However, in the reality, future short rates are not known.
- Nevertheless, it is still common to investigate the implications of the yield curve for future interest rates.

FORWARD INTEREST RATE

- Recognizing that future interest rates are uncertain, we call the interest rate that we infer in this manner the **forward interest rate** or the **future short rate**, denoted ${}_{t-1}f_t$ for period t , because it need not be the interest rate that actually will prevail at the future date.
- The sequence of forward rates for periods $t=1, \dots, T$ defines the **FORWARD YIELD CURVE**: $\{{}_1f_2, {}_2f_3, \dots, {}_{T-1}f_T\}$

FORWARD INTEREST RATE

- If the 1-year **forward rate** for period t is denoted ${}_{t-1}f_t$ we then define ${}_{t-1}f_t$ by the next equation:

$${}_{t-1}f_t = [(1+y_t)^t / (1+y_{t-1})^{t-1}] - 1$$

- Equivalently, we can express the spot rate for period t using the forward rates as follows:

$$(1+y_t)^t = (1+{}_0f_1)(1+{}_1f_2)\dots(1+{}_{t-1}f_t)$$

- Notice that ${}_0f_1 = y_1$.

SHORT AND FORWARD INTEREST RATES

- We emphasize that the interest rate that actually will prevail in the future **need not equal** the forward rate, which is calculated from today's data.
- Indeed, it is not even necessary the case that the forward rate equals the **expected value** of the future short rate.

SHORT AND FORWARD INTEREST RATES

- The difference between the forward rate and the future short rate is called liquidity premium.
- The name comes from the fact that the forward rate determines the interest that long-term investors obtain while short-term investors are more interested in the future short-rate.

SHORT AND FORWARD INTEREST RATES

- In order to relate the future short rates with forward rates, economists worked out several alternative theories.
- These are called **theories of term structure**. We will see two alternative theories:
 1. **Expectation hypothesis**
 2. **Liquidity preference theory**

EXPECTATION HYPOTHESIS

- The expectation hypothesis is the simplest theory of term structure.
- It states that the forward rate equals the market consensus expectation of the future short interest rate for all periods t :

$${}_{t-1}f_t = E[{}_{t-1}r_t]$$

LIQUIDITY PREFERENCE

- In this theory, we have two types of investors:
 1. **Short-term investors**: They prefer to invest for short time horizons.
 2. **Long-term investors**: They prefer to invest for long time horizons.

LIQUIDITY PREFERENCE

- **Short-term investors** are willing to hold long-term bonds if

$${}_{t-1}f_t > E[{}_{t-1}r_t]$$

- **Long-term investors** are willing to hold short-term bonds if

$${}_{t-1}f_t < E[{}_{t-1}r_t]$$

LIQUIDITY PREFERENCE

- People who believe in the liquidity preference theory think that **short-term investors dominate the market**.
- Therefore, in general the **forward rate is higher than the future short rate**:

$${}_{t-1}f_t > E[{}_{t-1}r_t]$$

- Therefore, the **liquidity premium** is positive:

$$\text{Liquidity premium} = {}_{t-1}f_t - E[{}_{t-1}r_t] > 0$$

LIQUIDITY PREFERENCE

- In the liquidity preference theory,
Forward rate = Short rate + Liquidity premium
- Therefore, the shape of the forward yield curve is determined by two components:
 1. Investors' expectations about future interest rates (short rate component) and
 2. Investors' future liquidity premium requirements (liquidity premium component).

FORWARD RATES AS FORWARD CONTRACTS

- We have seen how the forward rates can be derived from the spot yield curve.
- Why are these forward rates important from practical point of view?
- There is an important sense in which the forward rate is a market interest rate:

FORWARD RATES AS FORWARD CONTRACTS

- Suppose that you wanted to arrange **now** to make a loan at some **future** date.
- You would agree today on the interest rate that will be charged, but the loan would not commence until some time in the future.
- How would the interest rate on such a "forward loan" be determined?
- We will show that the interest rate of this forward loan would be the **forward rate**.

FORWARD RATES AS FORWARD CONTRACTS

Example:

- Suppose the price of 1-year maturity zero-coupon bond with face value EUR1000 is EUR952.38 and
- the price of 2-year zero-coupon bond with face value EUR1000 is EUR890.

FORWARD RATES AS FORWARD CONTRACTS

- We can determine two spot rates and the forward rate for the second period:

$$y_1 = 1000 / 952.38 - 1 = 5\%$$

$$y_2 = (1000 / 890)^{1/2} - 1 = 6\%$$

$${}_1f_2 = (1+y_2)^2 / (1+y_1) - 1 = 7.01\%$$

FORWARD RATES AS FORWARD CONTRACTS

- Now consider the following strategy:
 1. Buy one unit 1-year zero-coupon bond
 2. Sell 1.0701 unit 2-year zero-coupon bonds
- In the followings, we review the cash flows of this strategy:

FORWARD RATES AS FORWARD CONTRACTS

- At $t=0$, initial cash flow:
 1. Long one one-year zero:
 - 952.38 EUR
 2. Short 1.0701 two-year zeros:
 - $+890 \times 1.0701 = 952.38$ EUR
- TOTAL cash flow at $t=0$:
- $$- 952.38 + 952.38 = 0$$

FORWARD RATES AS FORWARD CONTRACTS

- At $t=1$, cash flow:
 1. Long one one-year zero:
 - +1000 EUR
 2. Short 1.0701 two-year zero:
 - 0 EUR
- TOTAL cash flow at $t=1$:
- $$+1000 \text{ EUR}$$

FORWARD RATES AS FORWARD CONTRACTS

- At $t=2$, cash flow:
 1. Long one one-year zero:
 - 0 EUR
 2. Short 1.0701 two-year zero:
 - $-1.0701 \times 1000 \text{ EUR} = - 1070.01$ EUR
- TOTAL cash flow at $t=2$:
- $$-1070.01 \text{ EUR}$$

FORWARD RATES AS FORWARD CONTRACTS

- In summary, we present the total cash flows over the two periods:

$t=0$	$t=1$	$t=2$
0 EUR	+1000 EUR	-1070.01 EUR
- Thus, we can see that the strategy creates a synthetic "forward loan": borrowing 1000 at $t=1$ and paying 1070.01 at $t=2$.
- Notice that the interest rate of this loan is

$$1070.01/1000 - 1 = 7.01\%$$
 which is equal to the **forward rate**.

FORWARD RATES AS FORWARD CONTRACTS

- Therefore, we can synthetically construct a forward loan by buying a shorter maturity zero-coupon bond and short selling a longer maturity zero-coupon bond.
- The interest rate of this forward loan is determined by the forward rate.

YIELD CURVE ESTIMATION

ESTIMATING THE YIELD CURVE

- We talked about how can we use the values of the spot yield curve in order to discount future cash flows.
- However, in the reality we do not observe the yield curve.
- In the real world, we observe:
 1. The bid and asked **prices** of bonds
 2. The **cash flows of coupon and par value payments** of bonds.

ESTIMATING THE YIELD CURVE

- It is useful from a practical point of view to **estimate** the spot yield curve because it helps us to discount cash flows paid at any time in the future.
- Therefore, given the yield curve we can price any fixed-income financial asset on the market.

ESTIMATING THE YIELD CURVE

- In this section, we review a methodology to estimate the spot yield curve given the observed bond prices and future cash flow payments.

ESTIMATING THE YIELD CURVE

- We will start with observed data on (1) bid and asked prices, (2) accrued interest and (3) future cash flows of several bonds traded on the market.
- We also know the exact day of each cash flow payment.
- In order to estimate the spot yield curve, we proceed as follows:

ESTIMATING THE YIELD CURVE

1. Compute the market price, p for each bond by the next formula:

$$p = (\text{Asked price} + \text{Bid price})/2 + \text{Accrued interest}$$

ESTIMATING THE YIELD CURVE

2. To get an estimate of the spot rate, y_t use the following **cubic polynomial** approximation of the log-spot rate, y_t :

$$\ln y_t = a + bt + ct^2 + dt^3$$

where a , b , c and d are the parameters of the cubic polynomial.

- **Remark 1:** We approximate the log-interest rate because we want to avoid sign restrictions on the a , b , c and d parameters (as y_t is positive).

ESTIMATING THE YIELD CURVE

- **Remark 2:** We employ a cubic polynomial approximation because a third-order polynomial can model the yield curve in a very flexible way:

- It can capture various types of increasing / decreasing / convex / concave parts of the yield curve.
- Therefore, the model can be very realistic.

ESTIMATING THE YIELD CURVE

- In the followings, first we assume that the parameter values are given and we present how to value of the bonds given these parameters estimates.
- Later, we shall discuss how can we estimate the parameters.

ESTIMATING THE YIELD CURVE

3. Given the parameters, we compute the value of y_t by taking the exponential of the cubic polynomial.
4. Then, we compute the discount factor for each point of time t according to the next formula:

$$DF(t, y_t) = \frac{1}{(1 + y_t)^t}$$

ESTIMATING THE YIELD CURVE

5. Afterwards, we use the discount factors to compute the present value of future cash flows: $PV(CF_t) = CF_t \times DF(t, y_t)$

6. Then, we sum these present values to get an estimate of the bond price:

$$p^* = \sum_{t=1}^T PV(CF_t)$$

where p^* is the bond price estimate, PV denotes present value.

ESTIMATING THE YIELD CURVE

7. Finally, we compute the following measure of estimation precision:

$$MSE = \frac{1}{N} \sum_{i=1}^N (p_i - p_i^*)^2$$

where MSE is the mean squared error, N is the number of bonds observed, p_i is the market price of the i -th bond and p_i^* is the estimate of the i -th bond price.

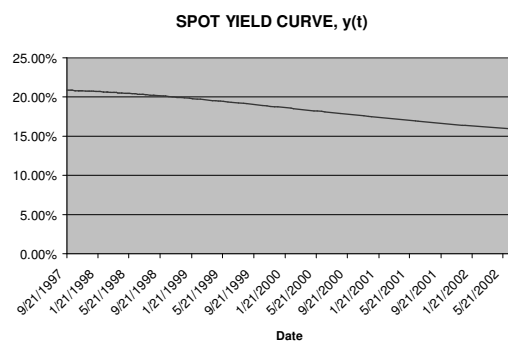
ESTIMATING THE YIELD CURVE

- How do we choose the values of the parameters?
- We choose parameter values such that the MSE precision measure is minimized.
- The MSE minimization can be done numerically in Excel using the **SOLVER** tool.
- (In Excel use: Tools / Solver or Herramientas / Solver.)

ESTIMATING THE YIELD CURVE

- The following figure show the spot yield curve estimate for Hungarian government bond data for the 1997-2002 period:

ESTIMATING THE YIELD CURVE



MANAGING BOND PORTFOLIOS

MANAGING BOND PORTFOLIOS

- We are going to review several topics related to bond portfolio management.
- In particular, we shall see:
 1. Evolution of bond prices over time
 2. Interest rate risk of bonds
 3. Default or credit risk of bonds

EVOLUTION OF BOND PRICES OVER TIME

EVOLUTION OF BOND PRICES OVER TIME

- In this section we are going to be in a dynamic framework.
- That is we shall analyze the evolution of the bond price over several periods:
- $t=0,1,2,\dots,T$.

EVOLUTION OF BOND PRICES OVER TIME

- As we have discussed before, the determinants of bond value are:
 1. **Future cash flow payments** (i.e., coupon payments and par value payments)
 2. Values of the **yield-to-maturity (YTM)** or **spot yield curve** used to discount these cash flows.

EVOLUTION OF BOND PRICES OVER TIME

- Future cash flows are **fixed in the contract**, therefore, they are **time invariant**. (Supposing that there is no default! In this section, we assume that there is no default risk. We shall see default risk later.)
- However, the YTM or the yield curve may **change over time**.

EVOLUTION OF BOND PRICES OVER TIME

- We shall investigate the evolution of bond price under two alternative situations:
 1. The YTM and spot yield curve are **constant** over time (NOT REALISTIC ASSUMPTION but it helps to understand a basic characteristic of the bond price evolution.)
 2. The YTM and spot yield curve are **change** over time (MORE REALISTIC SETUP)

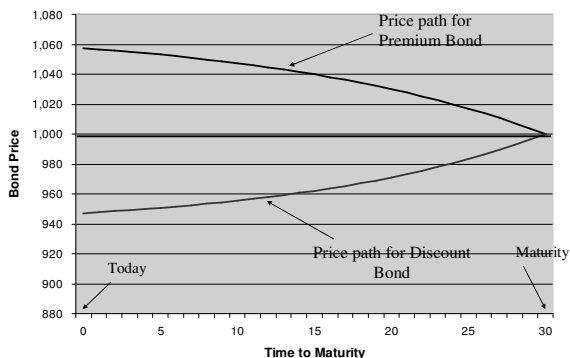
1. YTM and spot yield curve are constant

- When the market price of a bond is observed over several periods $t = 1, \dots, T$, we find that the **price of the bond is converging to its par value**.
- When we have a **premium bond** then the price of the bond is higher than the par value.
- Therefore, the bond price is decreasing during its convergence.

1. YTM and spot yield curve are constant

- On the other hand, when we have a **discount bond** then the price of the bond is lower than the par value.
- Therefore, the bond price is increasing during its convergence.
- The convergence of the bond price to its par value, under constant YTM, can be observed on the following figure:

1. YTM and spot yield curve are constant



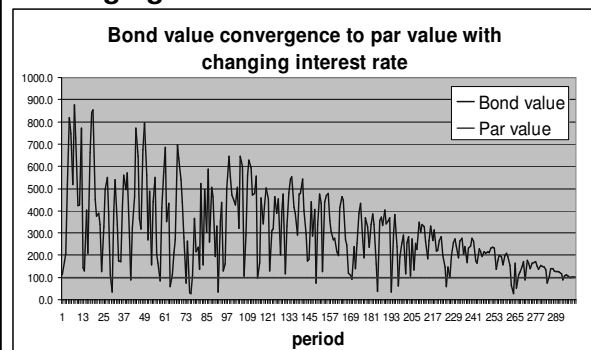
2. YTM and spot yield curve are changing

- In the reality, the level of the yield (YTM and spot yield curve) that is used to discount future cash flows is **not constant**.
- As the relation between the changing YTM and the fixed coupon rate may change, bonds may be discount or premium bonds over time.

2. YTM and spot yield curve are changing

- The following figure presents the evolution of bond price over time when YTM is changing over time:

2. YTM and spot yield curve are changing



2. YTM and spot yield curve are changing

- We obtained different yield for each period by Monte Carlo simulation from the following AR(1) process of log-return:

$$\ln r_t = c + \phi \ln r_{t-1} + \psi u_t$$

where c , ϕ and ψ are parameters and $u_t \sim N(0,1)$ i.i.d is the error term.

- After simulating $\ln r_t$ we take the exponential of it to get r_t .
- Note: We model log-return to ensure to the positivity of yield.

2. YTM and spot yield curve are changing

- As the yield is not constant, in some periods the bond value is higher than the par value and in other periods it is lower than the par value.
- However, in the figure we can see the convergence of the bond price to the par value as we approach to maturity.
- In the Excel spreadsheet, you can re-simulate the yield process with alternative parameters using the "F9" button.

INTEREST RATE RISK

INTEREST RATE RISK

- From the previous figure we can see that although bonds promise a fixed income payment over time, the actual price of a bond is affected by the level of interest rates.
- Therefore, fixed income securities are not risk-free.
- **Before the time of maturity**, their prices are volatile as they are impacted by the changing interest rate.

INTEREST RATE RISK

- The sensitivity of bond price to the interest rate is called **interest rate risk**.
- **Interest rate risk we only have before maturity** because the bond promises a fixed par value payment at maturity.
- The only case when the evolution of interest rates is important for the investor is when the investor wants to sell the bond before its maturity time.

INTEREST RATE RISK

- If an investor wants to avoid interest rate risk then it is enough to purchase a bond that will be held until the maturity time of the bond.
- By doing this, it is **not important** for the investor how the rates change during the lifetime of the bond.
- At maturity time, the investor will receive the fixed par value.

MANAGING INTEREST RATE RISK

- When an investor is interested in bond prices before the maturity time of bond then he is interested in the management of interest rate risk of his bond portfolio.
- A central concept of interest rate risk management is the **duration** and the **modified duration** of the bond portfolio.

DURATION

- The **duration** is the weighted average of the times of each coupon payment where the weights, w_t are

$$w_t = \frac{PV(CF)_t}{\text{Bond price}} = \frac{CF_t / (1 + y)^t}{\text{Bond price}}$$

where y is the YTM of the bond and **duration** is computed as

$$\text{Duration} = D = \sum_{t=1}^T t \times w_t$$

DURATION

- The duration can be interpreted as the effective **average maturity** of the bond portfolio.
- The scale of the duration is **years**.

Remarks about DURATION

- (1) The **duration** of a **zero-coupon bond** is **equal to the maturity** of the zero-coupon bond: $D=T$.
- (2) The **duration** of a T -period **annuity** is:
$$D=(1+y)/y - T / [(1+y)^T - 1]$$
- (3) The **duration** of a **perpetuity** is
$$D=(1+y)/y$$
where y is the yield of the annuity and perpetuity in (2) and (3).

MODIFIED DURATION

- The modified duration for any bond is defined as

$$\text{Modified duration} = D^* = \frac{D}{1+y}$$

where y is the YTM of the bond.

MODIFIED DURATION

- Modified duration can be used to compute the **interest rate sensitivity** of bond prices because:

$$\frac{\partial P}{\partial y} = -D^* P$$

where P is the bond price, y is the YTM

MODIFIED DURATION

- The **modified duration** also helps to answer the following more practical question:
- **Question:** What is the percentage change of the bond price when the interest rate changes by Δy ?
- **Answer:** When the interest rate change is relatively small than the percentage price change is approximately:

$$\frac{\Delta P}{P} \simeq -D^* \Delta y$$

MODIFIED DURATION

- **Remark:** Notice that if the duration (or modified duration) of a bond is higher then its interest rate sensitivity will be higher.
- In other words, bonds with longer maturity time are more sensitive to changes of the interest rate.
- In other words, **interest rate risk of long maturity bonds is higher than that of short maturity bonds**.

CONVEXITY

- The **convexity** of a bond is defined as

$$\text{Convexity} = \frac{1}{P(1+y)^2} \sum_{t=1}^T \left[\frac{CF_t}{(1+y)^t} (t^2 + t) \right]$$

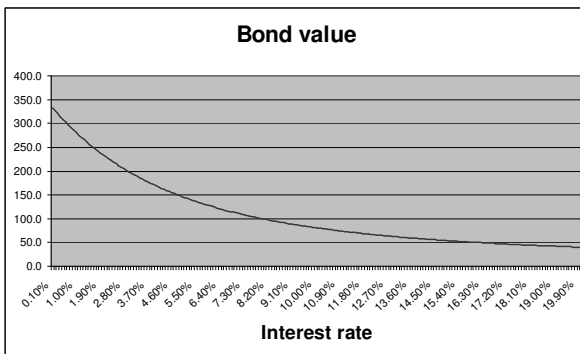
- **Convexity** is important because it is related to the second derivative of the bond:

$$\frac{\partial^2 P}{\partial y^2} = \text{Convexity} \times P$$

CONVEXITY

- In order to present this more clearly, where the “convexity” name comes from, we present the bond price as a function of the interest rate:

CONVEXITY



CONVEXITY

- Notice on this figure that the function of bond price, $P(y)$ is **convex**.
- This means that the shape of the curve implies that an increase in the interest rate results in a price decline that is smaller than the price gain resulting from a decrease of equal magnitude in the interest rate.

CONVEXITY

- As the $P(y)$ function is non-linear, the previously discussed

$$\frac{\Delta P}{P} \simeq -D^* \Delta y$$

formula is only an approximation of the percentage change of the bond price that only applies when the change of the interest rate is small.

CONVEXITY

- A more precise formula takes into account the convexity of the bond as well:

$$\frac{\Delta P}{P} = -D^* \Delta y + \frac{1}{2} \times \text{Convexity} \times (\Delta y)^2$$

- This formula applies when the change of the interest rate is large.
- It is also an approximation, however, it is more precise than the formula where only the first-derivative of $P(y)$ is included.

IMMUNIZATION

- Some financial institutions like banks or pension funds have fixed-income financial products in both the assets and liabilities sides of their balances.

IMMUNIZATION

- **Example:** A pension fund is receiving fixed payments from young clients who are working and paying every month the pension fund to get pension after their retirement. These payments are in the **asset** side of the balance.
- In the same time, the pension fund pays fixed monthly pensions to retired pensioners. These payments are on the **liability** side of the balance.

IMMUNIZATION

- These payments can be seen as bond portfolios.
- Therefore, they are subject to interest rate risk.
- How can a financial company manage the interest rate risk of its assets and liabilities?
- By doing **IMMUNIZATION**.

IMMUNIZATION

- Immunization means that the duration of assets and liabilities of the company are **equal**:

$$D_A = D_L$$

- When assets and liabilities of financial firms are **immunized** then a rate change has the same impact on its assets and liabilities.
- This where the name "immunization" comes from.

DEFAULT OF BONDS: CREDIT RISK

Default risk

- Although bonds generally promise a fixed flow of income, that income stream is not risk-free unless the issuer will not **default** on the obligation.
- While most government bonds may be treated as assets free of default risk, this is not true for corporate bonds.

Default risk

- Bond **default risk**, usually called **credit risk**, is measured by the next firms:
 1. **Moody's**,
 2. **Standard and Poor's (S&P's)** and
 3. **Fitch**

These institutions provide financial information on firms as well as quality ratings of large corporate and municipal bond issues.

Default risk

- International bonds, especially in emerging markets, also are commonly rated for default risk.
- Each rating firm assigns letter grades to the bonds to reflect their assessment of the safety of the bond issue.
- In the following table, the grades of Moody's and Standard and Poor's are presented:

Default risk

Moody's rating	S&P's rating	Quality of bond	Bond grade
Aaa	AAA	Very high quality	Investment-grade bond
Aa	AA	Very high quality	Investment-grade bond
A	A	High quality	Investment-grade bond
Baa	BBB	High quality	Investment-grade bond
Ba	BB	Speculative	Speculative-grade / Junk bond
B	B	Speculative	Speculative-grade / Junk bond
Caa	CCC	Very poor	Speculative-grade / Junk bond
Ca	CC	Very poor	Speculative-grade / Junk bond
C	C	Very poor	Speculative-grade / Junk bond
D	D	Very poor	Speculative-grade / Junk bond

Default risk

- At times Moody's and S&P's use adjustments to these ratings:
 1. S&P uses plus and minus signs: A+ is the strongest and A- is the weakest.
 2. Moody's uses a 1, 2 or 3 designation, with A1 indicating the strongest and A3 indicating the weakest.

Determinants of bond safety

- Rating agencies base their quality ratings largely on the level and trend of issuer's financial ratios.
- The key ratios are:
 1. **Coverage ratios:** Ratios of company earnings to fixed costs.
 2. **Leverage ratio:** Debt-to-equity ratio.

Determinants of bond safety

3. **Liquidity ratios:**
 - 3a. Current ratio =
Current assets / current liabilities
 - 3b. Quick ratio =
Current assets excluding inventories / current liabilities

Determinants of bond safety

4. Profitability ratios: Measures of rates of return on assets or equity

4a. Return on assets (ROA) =
Net income / total assets

4b. Return on equity (ROE) =
Net income / equity

5. Cash flow-to-debt ratio: Ratio of total cash flow to outstanding debt

Default premium

- To compensate for the possibility of default, corporate bonds must offer a **default premium**.
- The **default premium** is the difference between the promised yield on a corporate bond and the yield of an otherwise identical government bond that is risk-free in terms of default.

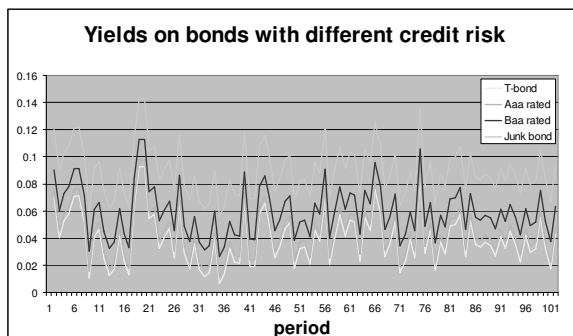
Default premium

- If the firm remains solvent and actually pays the investor all of the promised cash flows, the investor will realize a higher yield to maturity than would be realized from the government bond.
- However, if the firm goes bankrupt, the corporate bond will likely to provide a lower return than the government bond.
- That is why the corporate bond is riskier than the government bond.

Default premium

- The pattern of default premiums offered on risky bonds is sometimes called the **risk structure of interest rates**.
- The following figure shows the yield to maturity of different credit risk class bonds:

Default premium



DERIVATIVES

DERIVATIVES

A **derivative** is a financial instrument that is derived from some other asset (known as the **underlying asset**).

Rather than trade or exchange the underlying asset itself, derivative traders enter into an agreement that involves a final payoff which depends on the price of the underlying asset.

A GENERAL DEFINITION OF DERIVATIVES

- The final payoff of a derivative is always a specific function of past and current prices of the underlying asset.

$$(\text{Payoff of derivative at time } t) = f(S_1, \dots, S_t)$$

where S_t is the price of the underlying asset at time t and $f(\cdot)$ is the payoff function of the derivative.

A GENERAL DEFINITION OF RELATIVELY SIMPLE DERIVATIVES

- The payoff of the more simple derivatives depends only on the current price of the underlying asset:

$$(\text{Payoff of derivative at time } t) = f(S_t)$$

- Most derivatives that we see in this course have this type of payoff function.

A REMARK

- The word “derivative” has nothing to do with the derivative that you have studied in differentiation during the mathematical calculus course.
- The word “derivative” refers to the fact that the payoff of a derivative is a function of the price of the underlying asset.
- The payoff function defines the derivative.
- Different derivatives have different payoff functions.

DERIVATIVES

- Important types of derivatives are:
 1. **FUTURES and FORWARDS**
 2. **OPTIONS**
 3. **SWAPS**

FUTURES

FUTURES

- A contract to buy or sell an asset on a future date at a fixed price.
- The buyer and the seller of the futures contract have the obligation to buy or sell the asset.

FUTURES

- The buyer of the futures contract is in **long futures** position.
- The seller of the futures contract is in **short futures** position.

FUTURES

- The underlying asset of the futures contract can be:
 1. **Commodity** like grain, metals or energy
 2. **Financial product** like interest rate, exchange rate, stock or stock index

ELEMENTS OF FUTURES CONTRACT

- The main elements of the futures contract are the
 1. **Futures price, F**

This is the price fixed in the contract at which the transaction will occur in the future.
 2. **Expiration date, T**

This is the date fixed in the contract when the delivery will take place in the future.

PAYOFF OF FUTURES CONTRACT

- The payoff and the profit of the **long futures** position at the expiration date, T is

$$\text{Payoff} = \text{Profit} = S_T - F$$

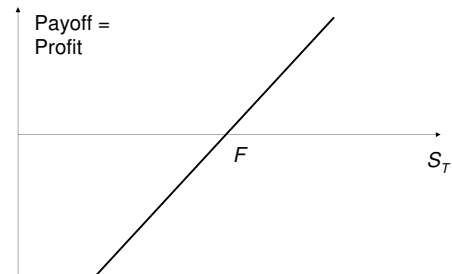
where S_T is the price of the underlying product at time T .

PAYOFF OF FUTURES CONTRACT

- Note that **payoff = profit** in the futures contract.
- This is because the contract is symmetric: both sides have obligation to buy/sell.
- Therefore, there is no cost of the establishment of the futures contract at time $t=0$.

PAYOFF OF FUTURES CONTRACT

- The payoff and profit of the **long futures** position can be presented on the next graph:



PAYOFF OF FUTURES CONTRACT

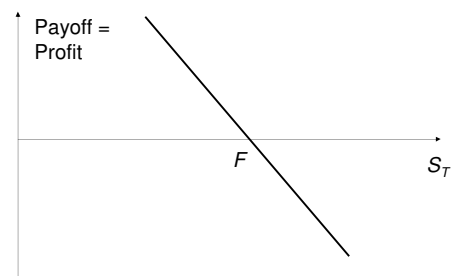
- The payoff and the profit of the **short futures** position at the expiration date, T is

$$\text{Payoff} = \text{Profit} = F - S_T$$

where S_T is the price of the underlying product at time T .

PAYOFF OF FUTURES CONTRACT

- The payoff and profit of the **short futures** position can be presented on the next graph:



FUTURES PRICE

Determining the correct futures price F :

Consider two alternative portfolios:

Portfolio 1:

- One **long futures** position of the underlying product with futures price F and maturity date T .
- One **risk-free treasury bill** (T-bill) with face value F and maturity date T . The T-bill pays risk-free rate of r .

FUTURES PRICE

Portfolio 2:

- One underlying product.

Payoffs at time $t=T$:

At time $t=T$, the T-bill will pay F amount of cash which will be used in the long futures contract to buy the underlying product at price F .

After buying the underlying using the LF contract both portfolios will be equal: both will have one underlying product.

SPOT-FUTURES PARITY

- Therefore, the cost of the establishment of both portfolios should be equal:
- Cost of portfolio 1 = $F/(1+r)^T = PV(F)$
- Cost of portfolio 2 = S_0
- Therefore, $F/(1+r)^T = S_0$
- And the correct futures price is given by
$$F = S_0(1+r)^T$$
- This equation is called **SPOT-FUTURES PARITY**.

SPOT-FUTURES PARITY with dividends

- A more general formulation of the **spot-futures parity** is obtained when the underlying product is a stock that pays dividend DIV until the maturity date T of the futures contract.
- The generalized spot-futures parity is given by:
$$F = S_0(1+r)^T - DIV = S_0(1+r-d)^T$$
where the second equality defines the **dividend yield**, d .

SPOT-FUTURES PARITY with dividends

Proof: Consider two alternative portfolios:

Portfolio 1:

- One **long futures** position of the underlying product with futures price F and maturity date T .
- One **risk-free treasury bill** (T-bill) with face value $(F+DIV)$ and maturity date T . The T-bill pays risk-free rate of r .

SPOT-FUTURES PARITY with dividends

Portfolio 2:

- One underlying product.

Costs and payoffs:

- Cost of establishment of the two portfolios at time $t=0$:

$$\text{Portfolio 1: } (F+DIV)/(1+r)^T$$

$$\text{Portfolio 2: } S_0$$

- Payoff of both portfolios at time $t=T$:
$$(S_T+DIV)$$

SPOT-FUTURES PARITY with dividends

- As the payoff is the same for both portfolios, the cost of establishment must be the same as well in order to avoid opportunities of arbitrage.
- Therefore,
$$(F+DIV)/(1+r)^T = S_0$$
and we get
$$F = S_0(1+r)^T - DIV$$

FORWARDS

FUTURES AND FORWARDS

- Forward contracts are the same as futures contracts:
- Both are about buying or selling an asset on a future date at a fixed price.
- In both, the buyer and the seller have the obligation to buy or sell the asset.
- Also the underlying asset of both contract can be either commodity or another financial asset.

FUTURES AND FORWARDS

- The distinction between “futures” and “forward” does not apply to the contract, but to **how the contract is traded**.

Trading of futures contracts

- Futures contracts are always traded in organized exchanges.
- In an organized exchange, futures products are standardized (with respect to possible maturity times and quality of products) and this way the **liquidity** of the futures market is increased.

Trading of futures contracts

- As futures products are standardized, it is possible that the quality and prices of “local” commodity product that the investor wants to hedge using a commodity futures contract is not the same as the quality and price of the underlying commodity of the futures contract traded at the organized exchange.

Trading of futures contracts

- Although there is a common dependence between local and exchange prices and quality (i.e. there is a high correlation), the correlation is not perfect.
- In risk management, this type of risk is called **basis risk**.

Trading of futures contracts

- Another consequence of standardized futures commodity exchanges is that the geographic location of the futures exchange may be far from the investor’s location.
- This can make costly and inconvenient the physical delivery of the commodity.

Trading of futures contracts

- Because of this reason, frequently, futures contracts are **closed** just before the maturity date and the corresponding profit or loss is delivered in cash.
- Closing a futures position means to open an opposite futures position to cancel the payoffs of both positions.
- For example, an investor having a LF position can close this by opening a SF position.

Trading of futures contracts

- When a futures contract is bought or sold, the investor is asked to put up a **margin** in the form of either cash or Treasury-bills to demonstrate that he has the money to finance his side of the bargain.
- In addition, futures contracts are **marked-to-market**. This means that each day any profit or losses on the contract are calculated and the investor pays the exchange any losses and receive any profits.

Trading of futures contracts

- For example, famous futures exchanges in the U.S. are:
 1. Chicago Mercantile Exchange Group (**CME Group**) that was formed by the fusion of Chicago Board of Trade (**CBOT**) and Chicago Mercantile Exchange (**CME**).
 2. New York Mercantile Exchange (**NYMEX**).

Trading of forward contracts

- Liquidity of futures exchanges is high because of standardization of the futures contracts.
- However, if the terms of the futures contracts do not suit the particular needs of the investor, he may be able to buy or sell **forward contracts**.

Trading of forward contracts

- The main forward market is in **foreign currency**. (**Forex market** or **FX market**)
- It is also possible to enter into a forward interest rate contract called **forward rate agreement** (FRA).

OPTIONS

OPTIONS

- An **option** is a contract between a buyer and a seller that gives the buyer the right - but not the obligation - to buy or to sell a particular asset (the **underlying asset**) at a later day at an agreed **strike price**.
- The purchase price of the option is called the **premium**. It represents the compensation the purchaser of the call must pay for the right to exercise the option.

OPTIONS

- Sellers of options, who are said to **write** options, receive premium income at the moment when the options contract is signed as payment against the possibility they will be required at some later date to deliver the asset in return for an **exercise price** or **strike price**.

CALL AND PUT OPTIONS

- A **call option** gives the buyer the right to **buy** the underlying asset.
- A **put option** gives the buyer of the option the right to **sell** the underlying asset.
- If the buyer chooses to exercise this right, the seller is **obliged** to sell or buy the asset at the agreed price.
- The buyer may choose not to exercise the right and let it expire.

OPTIONS POSITIONS

- The buyer of the **call option** is in **long call (LC)** position.
 - The seller of the **call option** is in **short call (SC)** position.
-
- The buyer of the **put option** is in **long put (LP)** position.
 - The seller of the **put option** is in **short put (SP)** position.

ELEMENTS OF OPTIONS CONTRACT

- The main elements of the options contract are the
 1. **Strike price, X**
This is the price fixed in the contract at which the buyer of the option can exercise his right to buy or sell the underlying product.
 2. **Expiration date, T**
This is the future date fixed in the contract until which the buyer can exercise his option.

EUROPEAN / AMERICAN OPTIONS

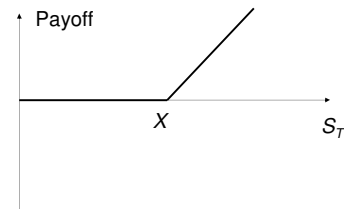
- There are two types of options:
 1. **European option:**
The buyer of the option can exercise his right to buy or sell the underlying product **only on the expiration date**.
 2. **American option:**
The buyer of the option can exercise his right to buy or sell the underlying product **at on or before the expiration date**.

PAYOFF OF OPTIONS

- In the following slides, we show the payoff and the profit of the call and put options.
- We shall use the following notation:
 1. **X**: strike price
 2. **T**: expiration date
 3. **S_T**: price of underlying product on the expiration date
 4. **c**: premium of the call option
 5. **p**: premium of the put option

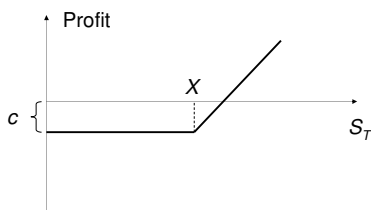
PAYOFF OF LONG CALL

- The **payoff** of the European long call position is
Payoff LC = max {S_T - X, 0}



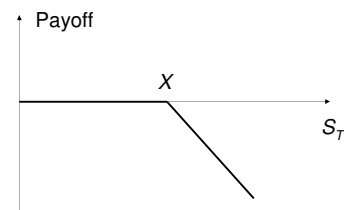
PROFIT OF LONG CALL

- The **profit** of the European long call position is
Profit LC = max {S_T - X, 0} - c



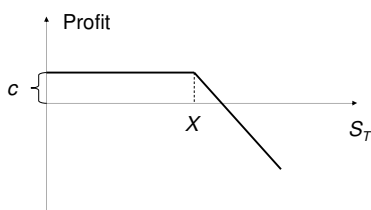
PAYOFF OF SHORT CALL

- The **payoff** of the European short call position is
Payoff SC = - max {S_T - X, 0}



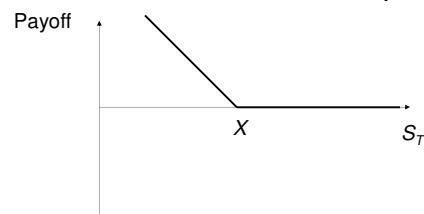
PROFIT OF SHORT CALL

- The **profit** of the European short call position is
Profit SC = - max {S_T - X, 0} + c



PAYOFF OF LONG PUT

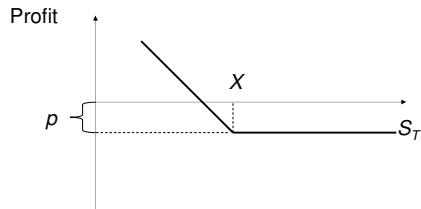
- The **payoff** of the European long put position is
Payoff LP = max {X - S_T, 0}



PROFIT OF LONG PUT

- The **profit** of the European long put position is

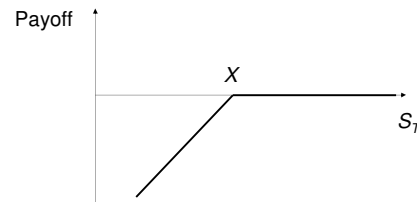
$$\text{Profit LP} = \max \{X - S_T, 0\} - p$$



PAYOFF OF SHORT PUT

- The **payoff** of the European short put position is

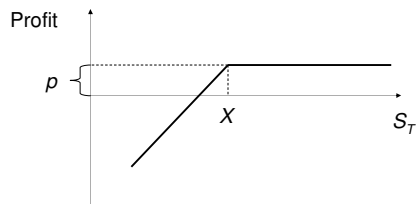
$$\text{Payoff SP} = - \max \{X - S_T, 0\}$$



PROFIT OF SHORT PUT

- The **profit** of the European short put position is

$$\text{Profit SP} = - \max \{X - S_T, 0\} + p$$



IN/AT/OUT OF THE MONEY OPTIONS

- An option is described as **in the money** when its exercise would produce positive payoff for its holder.
- An option is **out of the money** when exercise would produce zero payoff.
- Options are **at the money** when the exercise price and underlying asset price are equal.

UNDERLYINGS OF OPTIONS

Examples of options:

1. **Stock options:** the underlying product is a stock price.
2. **Index options:** the underlying product is a stock index.
3. **Futures options:** the underlying product is a futures contract.
4. **Foreign currency options:** the underlying product is an exchange rate.
5. **Interest rate options:** the underlying product is an interest rate

OPTIONS STRATEGIES

OPTIONS STRATEGIES

Options trading strategies:

1. Call options trading strategy
2. Put options trading strategy
3. Protective put strategy
4. Covered call strategy
5. Straddle strategy
6. Spread strategy

CALL OPTION STRATEGY

Call options trading strategy:

- Purchasing call options (LC) provide profit when the price of the underlying product increase.
- Selling call options (SC) provide profit when the price of the underlying product decrease.

PUT OPTION STRATEGY

Put options trading strategy:

- Purchasing put options (LP) provide profit when the price of the underlying product decrease.
- Selling put options (SP) provide profit when the price of the underlying product increase.

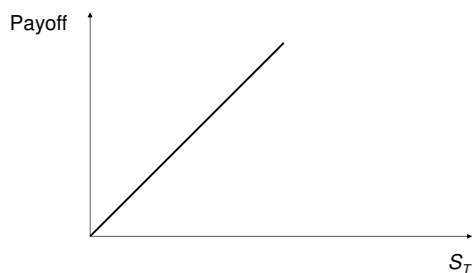
PROTECTIVE PUT STRATEGY

● Protective put strategy:

1. Buying the underlying product (Long Underlying) at price S_0 .
2. Buying a put option on the underlying product (LP) with strike price X .

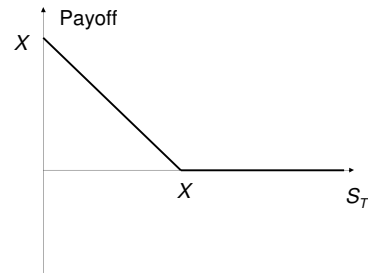
PROTECTIVE PUT STRATEGY

1. Payoff of the **long underlying** position:



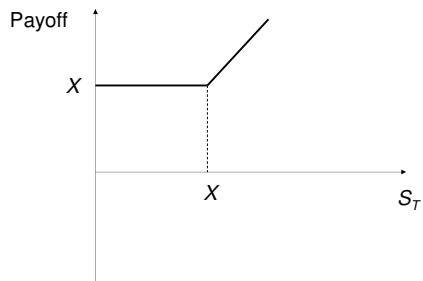
PROTECTIVE PUT STRATEGY

2. Payoff of the **LP position**:



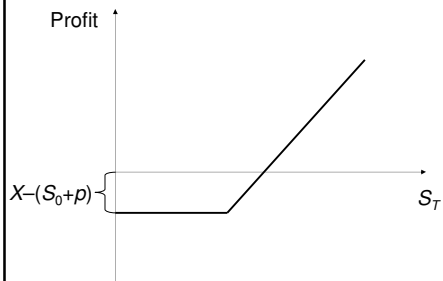
PROTECTIVE PUT STRATEGY

1+2. Payoff of the **protective put** strategy:



PROTECTIVE PUT STRATEGY

1+2. Profit of the **protective put** strategy:



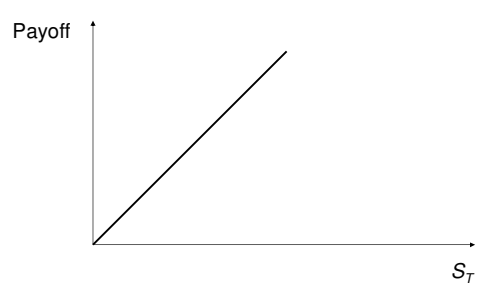
COVERED CALL STRATEGY

● **Covered call strategy:**

1. Purchase of the underlying product (long underlying position) at price S_0 .
2. Sale of a call option on the underlying (SC position) with strike price X .

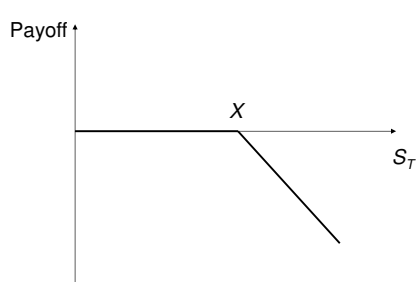
COVERED CALL STRATEGY

1. Payoff of the **long underlying** position:



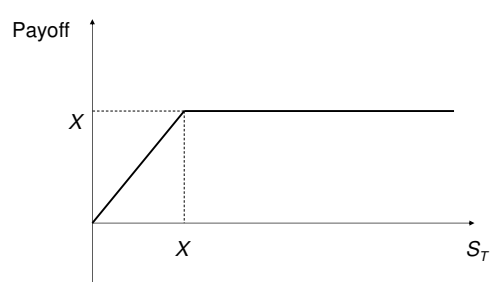
COVERED CALL STRATEGY

2. Payoff of the **SC** position:



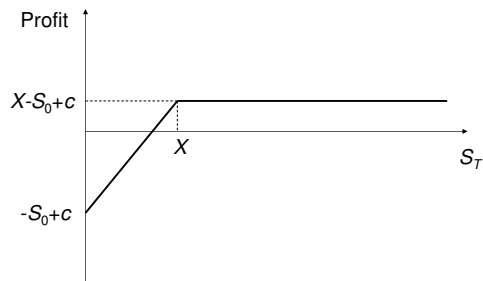
COVERED CALL STRATEGY

1+2. Payoff of the **covered call** strategy:



COVERED CALL STRATEGY

1+2. Profit of the **covered call** strategy:



STADDLE STRATEGY

● Straddle strategy:

1. Long straddle:

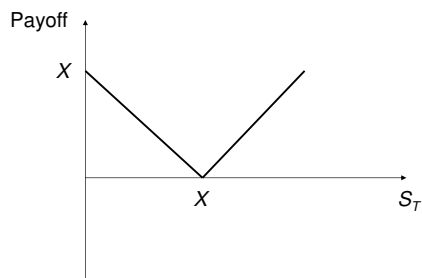
Buying both a call and a put option on the same underlying product each with the same strike price, X and expiration date, T .

2. Short straddle:

Selling both a call and a put option on the same underlying product each with the same strike price, X and expiration date, T .

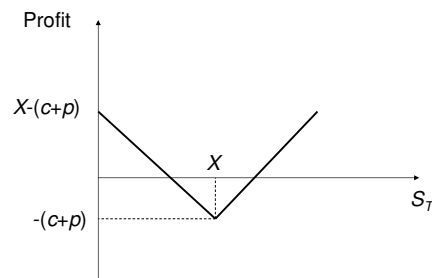
STADDLE STRATEGY

● Payoff of the long straddle:



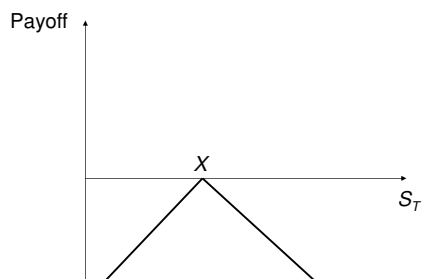
STADDLE STRATEGY

● Profit of the long straddle:



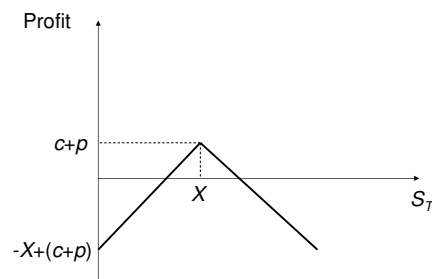
STADDLE STRATEGY

● Payoff of the short straddle:



STADDLE STRATEGY

● Profit of the short straddle:



STADDLE STRATEGY

Strip and strap strategies: These are variations of straddles.

1. **Long strip:** Buying two puts and one call with the same strike price and exercise date.
2. **Short strip:** Selling two puts and one call with the same strike price and exercise date.
3. **Long strap:** Buying two calls and one put with the same strike price and exercise date.
4. **Short strap:** Selling two calls and one put with the same strike price and exercise date.

SPREAD STRATEGY

- **Spread strategy:** A spread is a combination of two or more call options (or two or more put options) on the same underlying product with differing strike prices or expiration dates.
1. **Money spread:** involves the purchase and sale of options with different strike prices.
 2. **Time spread:** involves the purchase and sale of options with different expiration dates.

SPREAD STRATEGY

- In the followings, we shall focus only on money spreads.
 - We review three types of money spreads:
1. **BULLISH SPREAD:** used when the investor expects that the price of the underlying will increase.
 2. **BEARISH SPREAD:** used when the investor expects that the price of the underlying will decrease.
 3. **BUTTERFLY SPREAD:** used when the investor expects relatively small or relatively large price changes in the future.

SPREAD STRATEGY

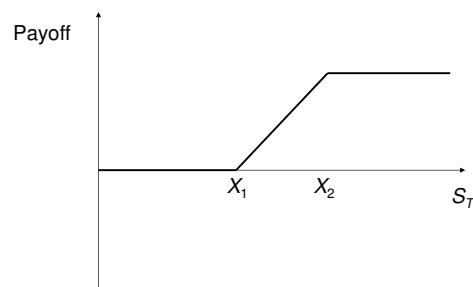
- **Bullish spread strategy:** This can be constructed in two alternative ways:
1. **First way:**
 - (1a) Buying a call option with strike price X_1 and
 - (1b) Selling a call option with strike price X_2 when $X_2 > X_1$.

SPREAD STRATEGY

2. **Second way:**
 - (2a) Buying a put option with strike price X_1 and
 - (2b) Selling a put option with strike price X_2 where $X_2 > X_1$.

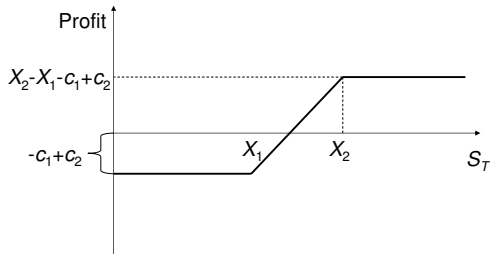
SPREAD STRATEGY

- Payoff of the **bullish spread:**



SPREAD STRATEGY

- Profit of the **bullish spread** (constructed from call options):



SPREAD STRATEGY

- **Bearish spread strategy:** This can be constructed in two alternative ways:

1. First way:

- (1a) Buying a call option with strike price X_1 and
- (1b) Selling a call option with strike price X_2 when $X_2 < X_1$.

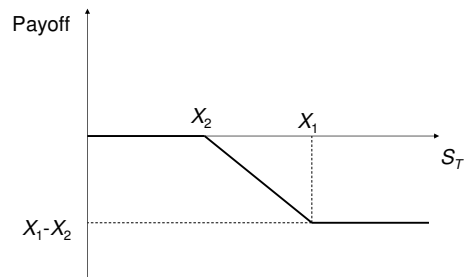
SPREAD STRATEGY

2. Second way:

- (2a) Buying a put option with strike price X_1 and
- (2b) Selling a put option with strike price X_2 where $X_2 < X_1$.

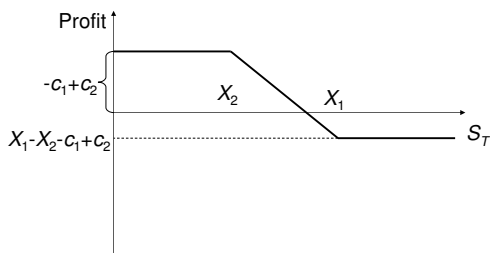
SPREAD STRATEGY

- Payoff of the **bearish spread**:



SPREAD STRATEGY

- Profit of the **bearish spread** (constructed from call options):



SPREAD STRATEGY

- The **butterfly spread strategy** has the following two types:

1. **LONG BUTTERFLY SPREAD**
2. **SHORT BUTTERFLY SPREAD**

SPREAD STRATEGY

1. LONG BUTTERFLY SPREAD

- It can be constructed in two alternative ways:

1. First way:

Purchase one call option with strike price X_1 .

Purchase one call option with strike price X_3 .

Sell two call options with strike price X_2 .

$X_1 < X_2 < X_3$ and $X_2 = (X_1 + X_3)/2$

SPREAD STRATEGY

2. Second way:

Purchase one put option with strike price X_1 .

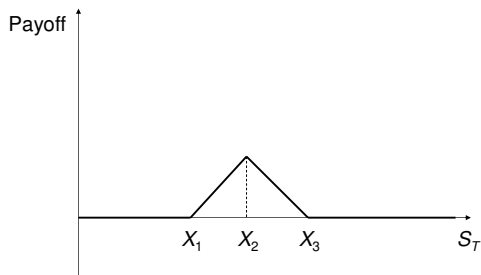
Purchase one put option with strike price X_3 .

Sell two put options with strike price X_2 .

$X_1 < X_2 < X_3$ and $X_2 = (X_1 + X_3)/2$

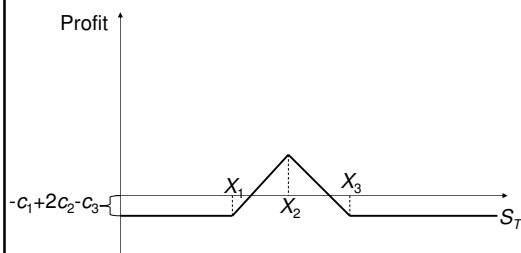
SPREAD STRATEGY

- Payoff of the long butterfly spread:



SPREAD STRATEGY

- Profit of the long butterfly spread (constructed from call options):



SPREAD STRATEGY

2. SHORT BUTTERFLY SPREAD

- It can be constructed in two alternative ways:

1. First way:

Sell one call option with strike price X_1 .

Sell one call option with strike price X_3 .

Buy two call options with strike price X_2 .

$X_1 < X_2 < X_3$ and $X_2 = (X_1 + X_3)/2$

SPREAD STRATEGY

2. Second way:

Sell one put option with strike price X_1 .

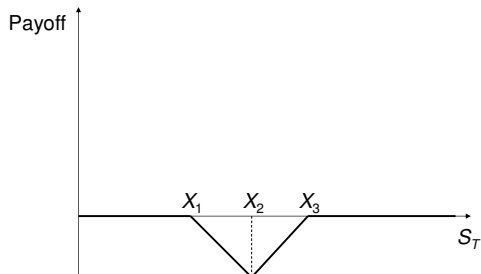
Sell one put option with strike price X_3 .

Buy two put options with strike price X_2 .

$X_1 < X_2 < X_3$ and $X_2 = (X_1 + X_3)/2$

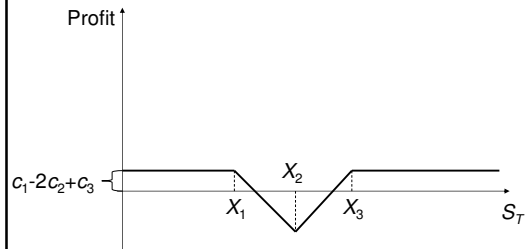
SPREAD STRATEGY

- Payoff of the **short butterfly spread**:



SPREAD STRATEGY

- Profit of the **short butterfly spread** (constructed from call options):



PUT-CALL PARITY

PUT-CALL PARITY

- The put-call parity is an important formula because it establishes the relationship between the prices of call and put options, the underlying product and the risk-free bond.
- The put-call parity must hold on the financial market in order to avoid arbitrage opportunities.
- On the following slides, the put-call parity is derived.

PUT-CALL PARITY

- Consider two alternative portfolios established at time $t=0$:

Portfolio 1:

- Buy one **European call option** with strike price X and expiration date T .
- Buy one **risk-free treasury bill** (T-bill) with face value X and maturity date T .

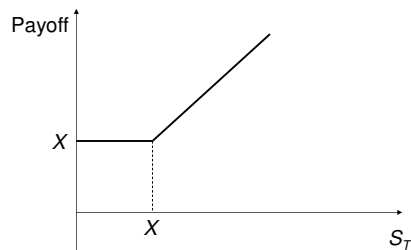
PUT-CALL PARITY

Portfolio 2:

- Buy one **European put option** with strike price X and expiration date T and
- Buy one **underlying product**.

PUT-CALL PARITY

- Notice that the payoff of both portfolios at time T is equal independently of S_T :



PUT-CALL PARITY

- If the payoff of the portfolios is equal at time T then the cost of establishment of the two portfolios at time $t=0$ should be equal as well.
- The cost of Portfolio 1 = $c + PV(X) = c + X/(1+r)^T$
- The cost of Portfolio 2 = $p + S_0$

PUT-CALL PARITY

- The consequence is that $c + X/(1+r)^T = p + S_0$
or
 $c + PV(X) = p + S_0$
- This equation explains the relationship between the prices of the call and put options and is called **PUT-CALL PARITY**.

PUT-CALL PARITY with dividends

- More general formulation of the **put-call parity for dividend paying stocks**:
- Suppose that the underlying product is a stock that pays dividends, DIV until the expiration date T .
- Then, we can reformulate the put-call parity as follows:
 $c + PV(X) + PV(DIV) = p + S_0$

PUT-CALL PARITY with dividends

Proof:

- Consider two alternative portfolios at time $t=0$:

Portfolio 1:

- Buy one **European call option** with strike price X and expiration date T .
- Buy one **risk-free treasury bill** (T-bill) with face value $(X+DIV)$ and maturity date T .

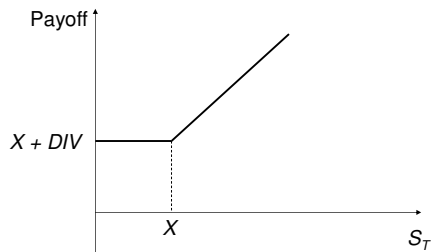
PUT-CALL PARITY with dividends

Portfolio 2:

- Buy one **European put option** with strike price X and expiration date T and
- Buy one **underlying product**.

PUT-CALL PARITY with dividends

- Notice that the payoff of both portfolios at time T will be equal independently of S_T :



PUT-CALL PARITY with dividends

- If the payoff of the portfolios is equal at time T then the cost of establishment of the two portfolios at time $t=0$ should be equal too.
- The cost of Portfolio 1 =
 - $c + PV(X+DIV) = c + (X+DIV)/(1+r)^T$
- The cost of Portfolio 2 =
 - $p + S_0$

PUT-CALL PARITY with dividends

- The consequence is that
$$c + (X+DIV)/(1+r)^T = p + S_0$$
or
$$c + PV(X) + PV(DIV) = p + S_0$$
- This is the put-call parity for a dividend paying stock.

EXOTIC OPTIONS

EXOTIC OPTIONS

EXOTIC OPTIONS:

1. Bermuda option
2. Compound option
3. Chooser option
4. Barrier option
5. Binary option
6. Lookback option
7. Asian option

Bermuda option

- The **Bermuda option** is similar to the American option. That is it can be exercised on dates before the date of exercise.
- However, unlike to American option that can be exercised on **any date** before or on the exercise date, the Bermuda option can be exercised **only on a limited number of dates** before the exercise date.

Compound option

- The compound option is an option whose underlying product is an option.
- There are four types of compound option:
 1. Call option on a call option (underlying = call option)
 2. Put option on a call option (underlying = call option)
 3. Call option on a put option (underlying = put option)
 4. Put option on a put option (underlying = put option)

Chooser option

- In the “chooser” or “as you like it” option, the buyer of the option can choose between having a **call option** OR a **put option** after buying the option.

Barrier option

- **Barrier options** have payoffs that depend not only on some asset price on the expiration date, but also on whether the underlying asset price has crossed through some “barrier”.
- A barrier option is a type of option where the option to exercise depends on the underlying crossing or reaching a given barrier level.

Barrier option

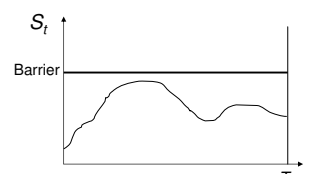
- Barrier options are always **cheaper** than a similar option without barrier.
- Therefore, barrier options were created to provide the insurance value of an option **without charging as much premium**.

Barrier option

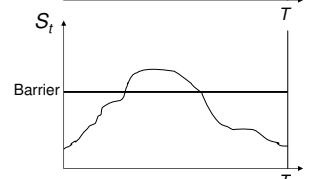
There are four types of barrier options:

1. **Up-and-out**: the price of the underlying starts below the barrier level and has to move up to the barrier level to be knocked out.
2. **Down-and-out**: the price of the underlying starts above the barrier level and has to move down to the barrier level to be knocked out.

Up-and-out barrier call option

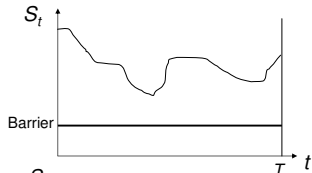


Payoff:
 $\max \{S_T - X, 0\}$

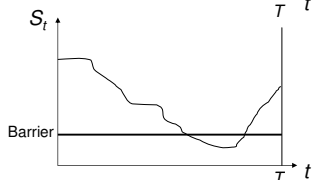


Payoff:
 As the barrier has been crossed before $t=T$, the call option has been knocked out thus its payoff is zero.

Down-and-out barrier call option



Payoff:
 $\max \{S_T - X, 0\}$

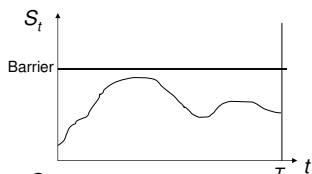


Payoff:
 As the barrier has been crossed before $t=T$, the call option has been knocked out thus its payoff is zero.

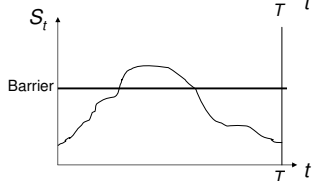
Barrier option

- Up-and-in:** the price of the underlying starts below the barrier level and has to move up to the barrier level to become activated.
- Down-and-in:** the price of the underlying starts above the barrier level and has to move down to the barrier level to become activated.

Up-and-in barrier call option

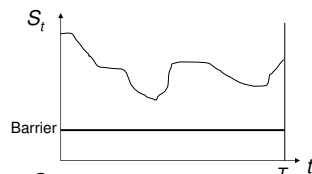


Payoff:
 As the barrier has not been crossed before $t=T$, the call option has not become activated thus its payoff is zero.

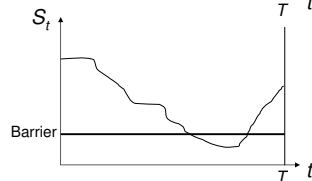


Payoff:
 The barrier has been passed thus the option has been activated:
 $\max \{S_T - X, 0\}$

Down-and-in barrier call option



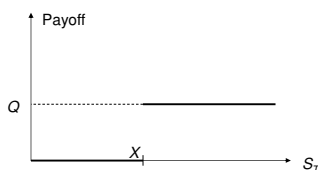
Payoff:
 As the barrier has not been crossed before $t=T$, the call option has not become activated thus its payoff is zero.



Payoff:
 The barrier has been passed thus the option has been activated:
 $\max \{S_T - X, 0\}$

Binary option

- The **binary option** is an option with discontinuous payoff.
- An example of the binary option is the “**cash-or-nothing call**”. This option pays nothing if $S_T < X$ and pays a fixed cash Q if $S_T \geq X$.



Lookback option

- Lookback options** have payoffs that depend in part on the **minimum** or **maximum** price of the underlying asset during the life of the option.
- For example, the payoff of a lookback call option may depend on the maximum price:

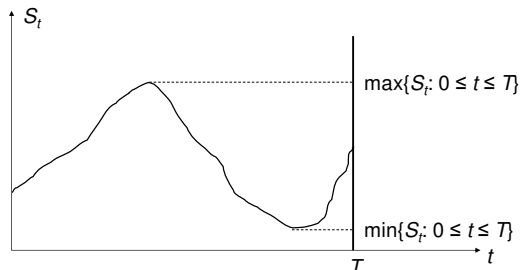
$$\text{Payoff} = \max\{\max\{S_i\} - X, 0\}$$

or the minimum price of the underlying asset:

$$\text{Payoff} = \max\{\min\{S_i\} - X, 0\}$$

Lookback option

- The payoff of lookback options depends on the evolution of the price of the underlying product during $0 \leq t \leq T$.



Asian option

- Asian options are options with payoffs that depend on the **average price** of the underlying asset during at least some portion of the life of the option.

- For example, the payoff of an Asian call option can be

$$\text{Payoff} = \max\{\text{mean}(S_t) - X, 0\}$$

where $\text{mean}(S_t)$ is the average price of the underlying during the lifetime of the option.

PRICING DERIVATIVES

ASSET PRICING

- First, we give a short introduction of two alternative asset pricing approaches of finance:
 1. **Expectation pricing** and
 2. **Arbitrage pricing**
- Then, we present two alternative approaches of derivatives pricing:
 1. **Binomial tree approach** and
 2. **Black-Scholes model**

Expectation pricing models

1. Expectation pricing models

- **Expectation pricing** models use several assumptions regarding investors' preferences and solve expected utility maximization problems to derive prices.
- In expectation pricing, we need to assume a **distribution** for future **returns** because we maximize the **expected value** of random returns of investments.

1. Expectation pricing models

- This assumption may fail easily and thus the prices obtained by expectation pricing are not robust in general.
- Expectation pricing **does not enforce** market prices, it only gives a **suggestion** for market prices.
- A famous equilibrium pricing model is the **capital asset pricing model (CAPM)**.

Arbitrage pricing models

2. Arbitrage pricing models

- An alternative approach is **arbitrage pricing**, where we do not assume anything about the '*real world*' probability distribution of future returns.
- Arbitrage pricing is **more frequently used** in practice than expectation pricing.
- Arbitrage pricing enforces market prices therefore it is a **more robust** pricing result.

Arbitrage

- **Definition** (*arbitrage opportunity*): An arbitrage opportunity arises when the investor can construct a zero investment portfolio that will yield a **sure** profit.
- In other words, the exploitation of security mispricing in such a way that risk-free economic profits may be earned is called arbitrage.

Arbitrage

- It involves the simultaneous purchase and sale of equivalent securities in order to profit from discrepancies in their price relationship, and so it is an extension of the law of one price.
- The concept of arbitrage is central to the theory of financial markets.

Arbitrage

- **Assumption:** In order to be able to construct a zero investment portfolio, one has to be able to sell short at least one asset and use the proceeds to purchase (to go long on) one or more assets.
- Borrowing may be considered as a short position in the risk-free asset.
- Even a small investor using short positions can take a large dollar / euro position in such a portfolio.

Arbitrage

- A critical property of a risk-free arbitrage portfolio is that any investor, regardless of risk aversion or wealth, will want to take an infinite position in it so that profits will be driven to an infinite level.
- Because those large positions will force prices up or down until the opportunity vanishes, we can derive restrictions on security prices that satisfy the condition that no arbitrage opportunities are left in the marketplace.

Difference between arbitrage and expectation arguments

- Expectation pricing builds on investors' opinion about future returns, while arbitrage pricing uses the price discrepancies among different assets.
- Therefore, we may say that expectation pricing leads to "**absolute prices**" and arbitrage pricing derives "**relative prices**".

Difference between arbitrage and expectation arguments

- There is another important difference between arbitrage and expectation arguments in support of **equilibrium price relationships**.
- When an expectation argument holds on the market and the equilibrium price relationship is violated, many investors will make portfolio changes.

Difference between arbitrage and expectation arguments

- In an expectation pricing model, each individual investor will make a limited change, though, depending on his or her degree of risk aversion.
- Aggregation of these limited portfolio changes over many investors is required to create a large volume of buying or selling, which in turn restores equilibrium prices.

Difference between arbitrage and expectation arguments

- However, when arbitrage opportunities exist, each investor wants to take as large position as possible.
- Therefore, it will not take many investors to bring about price pressures necessary to restore equilibrium.
- For this reason, implications for prices derived from no-arbitrage arguments are stronger than implications derived from a risk-versus-return dominance argument.

DERIVATIVES PRICING

PRICING DERIVATIVES

- Derivatives are usually priced by arbitrage pricing models in practice.
- In this section, we present two alternative pricing approaches used for derivatives:
 1. **Binomial tree approach of Cox-Ross-Rubinstein.**
 2. **Black-Scholes formula**

BINOMIAL APPROACH

BINOMIAL APPROACH

- The binomial approach:
 - (1) Applies to derivatives with **different payoff functions**. (For example, it can be applied to price some exotic derivatives.)
and
 - (2) **Does not assume** any particular probability structure for the evolution of the price of the underlying. (We do not assume anything about the **probability of price increase or price decrease**.)

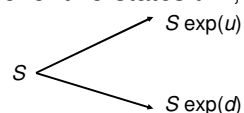
BINOMIAL APPROACH

- Binomial derivatives pricing is in **discrete time**: Financial transactions and payoffs occur at discrete points of time $t=1,2,\dots,T$.
- We present the binomial approach in two steps:
 1. Two-state framework: $t=1,2$
 2. Multi-state framework: $t=1,2,\dots,T$

TWO-STATE FRAMEWORK

TWO-STATE FRAMEWORK

- Suppose that the **price** of the **underlying asset** moves along the following binomial tree over two-states $t=1,2$:



where $u>0$ and $d<0$ are the log-returns of the underlying asset over the two states:

$$\text{log-return} = \ln \left[\frac{S \exp(u)}{S} \right] = \ln[\exp(u)] = u$$

A REMARK ABOUT DISCOUNTING

- There are two alternative definitions of the discount factor that we use in this course:

(1) DF that uses “traditional returns”:

$$DF(r, T) = \frac{1}{(1+r)^T}$$

(2) DF that uses “log-returns”:

$$DF(r, T) = \exp(-rT)$$

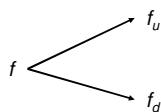
- In the derivatives pricing section, we use (2) to discount future cash flows.

TWO-STATE FRAMEWORK

- In the binomial tree approach, we **do not need to assume anything** about the probability of price increase or price decrease.
- The only assumption we make about the evolution of S is that it goes along a binomial tree. (That is in every period it goes up or down with certain log-return u or d .)

TWO-STATE FRAMEWORK

- We are interested in determining the price, f of a derivative whose payoff is a function of the price of the underlying asset:



where f is the price of the derivative at $t=1$ and f_u and f_d are the payoffs of the derivative at $t=2$.

TWO-STATE FRAMEWORK

- In order to price the derivative, consider the following portfolio:

1. Buy Δ units of the underlying asset
2. Sell 1 unit of the derivative

- If the price of the underlying goes up then the value of this portfolio at $t=2$ is

$$\Delta S \exp(u) - f_u$$

otherwise the value of this portfolio at $t=2$ is

$$\Delta S \exp(d) - f_d$$

TWO-STATE FRAMEWORK

- First, we determine the value of Δ that makes the portfolio risk-free.
- The portfolio is risk-free when its value is the same either S goes up or down:

$$\Delta S \exp(u) - f_u = \Delta S \exp(d) - f_d$$

- From this equation, we get the number of underlying assets that we need to buy to have a risk-free portfolio:

$$\Delta = [f_u - f_d] / [S \exp(u) - S \exp(d)] \quad (1)$$

TWO-STATE FRAMEWORK

- As the portfolio is risk-free, the price of the portfolio at $t=1$ is equal to the present value of its payoff at $t=2$ computed using the risk-free rate, r :

$$\Delta S - f = \exp(-r) [\Delta S \exp(u) - f_u]$$

- From this equation, we get the **price of the derivative**, f :

$$f = \Delta S - \exp(-r) [\Delta S \exp(u) - f_u] \quad (2)$$

TWO-STATE FRAMEWORK

- We can get an alternative formula for the **price of the derivative**, f in the following way:

- Substitute (1) into (2). Then, we get:

$$f = \exp(-r) [p f_u + (1-p) f_d] \quad (3)$$

where p is defined as

$$p = \frac{\exp(r) - \exp(d)}{\exp(u) - \exp(d)} \quad (4)$$

TWO-STATE FRAMEWORK

- **Suppose that $d \leq r \leq u$.** (This is a reasonable assumption because d is negative, r is the risk-free rate and u is the rate of the risky underlying asset.)
- Then, it follows from equation (4) that $0 \leq p \leq 1$.
- Therefore, p can be interpreted as a **probability**.

TWO-STATE FRAMEWORK

- If we interpret p in equation (4) as the probability that the price of S goes up then equation (3) has a clear meaning:
- **The price of the derivative is the present value, discounted by the risk-free rate, of the expected payoff of the derivative, where the expected value is computed using the probabilities p and $(1-p)$.**

TWO-STATE FRAMEWORK

- This pricing approach is very important in derivatives pricing.
- It is called **risk-neutral pricing** and the probability p is called **risk-neutral probability**.

TWO-STATE FRAMEWORK

- **Remark:** Risk neutral pricing has nothing to do with expectation pricing because in risk neutral pricing we use **risk neutral probabilities** to compute the expected payoff of the derivative whereas in expectation pricing we use the **real world probabilities** for the same computation.

TWO-STATE FRAMEWORK

- It is important to understand that the price of the derivative we have already obtained in equation (2) **before** introducing the concept of the risk-neutral probability.
- Then, why did we introduce risk-neutral probability?
- Because equation (3) has a more clear and intuitive meaning than (2): **The price of the derivative is the present value of its expected payoff.**

TWO-STATE FRAMEWORK

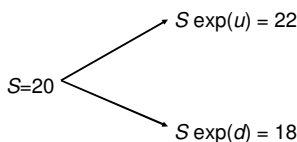
- Nevertheless, the present value is computed using the risk-free rate and the expected value is computed using the risk-neutral probability.
- Thus, when we use equation (3) to compute the price f then we are in an *artificial world* called the '**risk-neutral world**'.

TWO-STATE FRAMEWORK

- The '*risk-neutral world*' is nothing else but a pure mathematical construction where we travel to price derivatives.
- However, the derivative pricing problem has essentially **nothing to do with the probability of true world events**. It is nothing else but a problem of **linear algebra**.
- That is why it is **not necessary** to know the probability of the **true world** that the price of S goes up or down.

EXAMPLE

- Compute the price of a call option on a stock with strike price $X=21$ if
- The risk-free rate is 3% and
- The evolution of the stock price is according to the following tree:



EXAMPLE

- First, write the payoff of the call option as a function of the stock price:

$$f \begin{cases} f_u = \max\{S \exp(u) - X, 0\} = \max\{22-21, 0\} = \max\{1, 0\} = 1 \\ f_d = \max\{S \exp(d) - X, 0\} = \max\{18-21, 0\} = \max\{-3, 0\} = 0 \end{cases}$$

EXAMPLE

- Second, compute $\exp(u)$ and $\exp(d)$ from the tree of the stock price:
 - $S \exp(u) = 22$
 - $20 \exp(u) = 22$
 - $\exp(u) = 22/20$**
 - $S \exp(d) = 18$
 - $20 \exp(d) = 18$
 - $\exp(d) = 18/20$**

EXAMPLE

- Third, compute the value of Δ :

$$\Delta = [f_u - f_d] / [S \exp(u) - S \exp(d)] = [1 - 0] / [22 - 18] = 0.25$$
- Δ is interpreted as follows:
- For each unit of the call option need to buy 0.25 units of the stock in order to make the portfolio (1) 1 unit short call and (2) 0.25 units long underlying risk-free.

EXAMPLE

- Next, we compute the value of the call option using formula (2):

$$f = \Delta S - \exp(-r) [\Delta S \exp(u) - f_u] =$$
$$0.25 \times 20 - \exp(3\%) \times [0.25 \times 22 - 1] =$$
$$5 - 4.367 = \mathbf{0.6330}$$

- So the price of the call option is **0.6330**.

EXAMPLE

- Alternatively, we can compute the price of the call option using the risk-neutral pricing of equation (3):

$$f = \exp(-r) [p f_u + (1-p) f_d]$$

- To do this this, first, we need to compute p :

$$p = [\exp(r) - \exp(d)] / [\exp(u) - \exp(d)] =$$
$$[\exp(3\%) - 18/20] / [22/20 - 18/20] =$$
$$0.1304 / 0.2 = 0.652$$

EXAMPLE

- Now, we can compute equation (3):

$$f = \exp(-r) [p f_u + (1-p) f_d] =$$
$$\exp(-3\%) \times [0.652 \times 1 + (1-0.652) \times 0] =$$
$$0.6330$$

- So we get the same price for the call option as before: **0.6330**.

MULTI-STATE FRAMEWORK

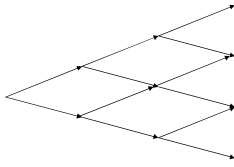
MULTI-STATE FRAMEWORK

- In the two-state methodology presented before we assumed that:
- There are only two-states $t=1,2$ and that
- There are only two possible outcomes of the price of the underlying asset (up or down).
- However, it would be more realistic if we would have T -states $t=1,2,\dots,T$ and more outcomes for the price of the underlying.

MULTI-STATE FRAMEWORK

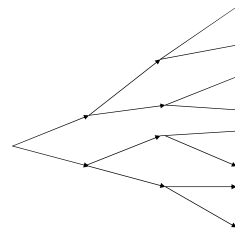
- Therefore, we generalize the previous two-step framework to a multi-step setup.
- We may decide to model the price process of the underlying asset according to the following two trees:
 1. Recombining tree
 2. Non-recombining tree

MULTI-STATE FRAMEWORK RECOMBINING TREE



Notice that on this figure $t=1, \dots, 4$ and the number of possible outcomes is 4.

MULTI-STATE FRAMEWORK NON-RECOMBINING TREE

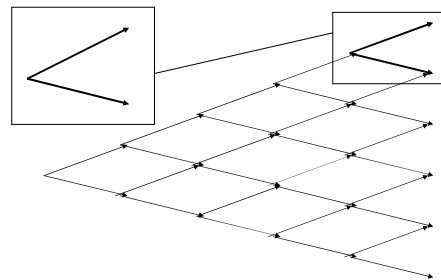


Notice that on this figure $t=1, \dots, 4$ and the number of possible outcomes is 8.

MULTI-STATE FRAMEWORK

- We can see that the non-recombining tree allows more outcomes, i.e. it is more general.
- The pricing approach to be discussed applies to both types of trees.
- However, it is computationally easier to work with the recombining tree.
- In the followings, we focus on the recombining tree to compute prices.

MULTI-STATE FRAMEWORK

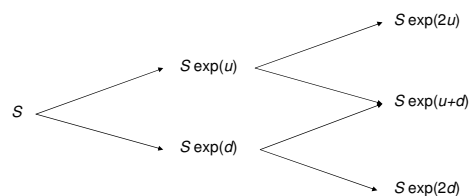


Notice that the multi-state tree is simply the sum of two-state trees.

MULTI-STATE FRAMEWORK

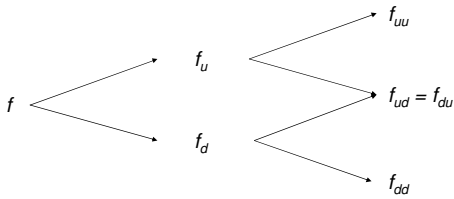
- If we think of the multi-state tree as the sum of several two-state trees then we can easily apply the approach introduced in the two-state slides for the multi-state tree.
- To show how to do it in practice we consider three states:

MULTI-STATE FRAMEWORK



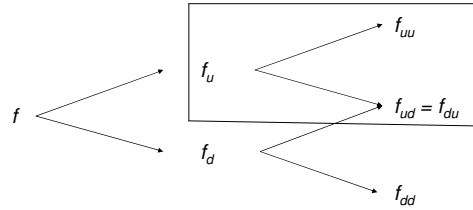
- This is the tree of prices of the underlying asset.

MULTI-STATE FRAMEWORK



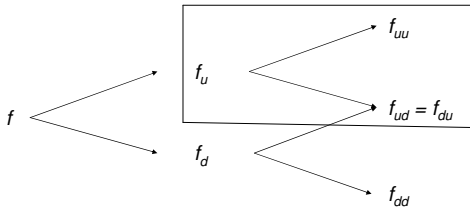
- This is the tree of prices and payoffs of the derivative to be priced.

MULTI-STATE FRAMEWORK



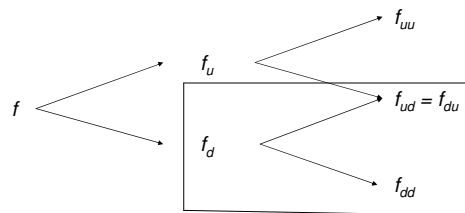
- The objective is to determine the value f .
- We do it **backward**: we start with the last nodes of the tree. (See the upper right box on the graph).

MULTI-STATE FRAMEWORK



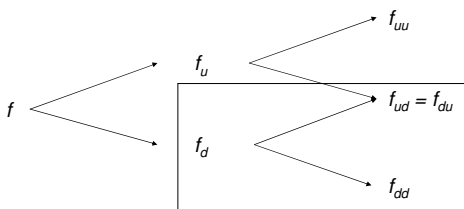
- In the first step, we determine the payoffs f_{uu} and f_{ud} of the last state.
- Then, we use the two-state framework to compute the value f_u .

MULTI-STATE FRAMEWORK



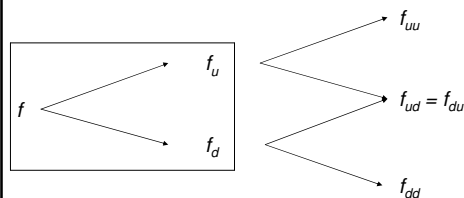
- Then, we focus on the lower right rectangle of the figure. (See on the graph.)

MULTI-STATE FRAMEWORK



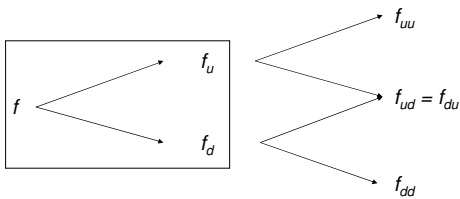
- In the second step, we determine the payoffs f_{ud} and f_{dd} of the last state.
- Then, we use the two-state framework to compute the value f_d .

MULTI-STATE FRAMEWORK



- Once we have computed f_u and f_d , we focus on the final rectangle presented on the graph.

MULTI-STATE FRAMEWORK



- In the third step, we compute f using the two-stage framework.

MULTI-STATE FRAMEWORK

- The approach presented for $t=1,2,3$ is straightforward to extend to any $t=1,\dots,T$.
- It can be applied for recombining and for non-recombining trees as well.
- **It can be applied to price many financial derivatives** (derivatives with complicated payoffs).
- Therefore, it is a quite general approach of derivatives pricing.

BLACK-SCHOLES FORMULA

BLACK-SCHOLES FORMULA

- The **Black-Scholes (BS) formula** is one of the most applied formulas in finance.
- It is applied daily in order to price derivatives in financial markets.
- It was developed by Fisher Black, Myron Scholes and Robert Merton.
- Fisher Black died in 1995.
- In 1997, Scholes and Merton received Nobel Prize in Economics.

BLACK-SCHOLES FORMULA

- The BS formula is used to price European call or European put options.
- The BS model assumes that the price of the underlying asset, S follows a continuous time '*geometric Brownian motion*':

$$dS = \mu S dt + \sigma S dz$$

where μ and σ are two parameters of the price process of S .

BLACK-SCHOLES FORMULA

The differential equation

$$dS = \mu S dt + \sigma S dz$$

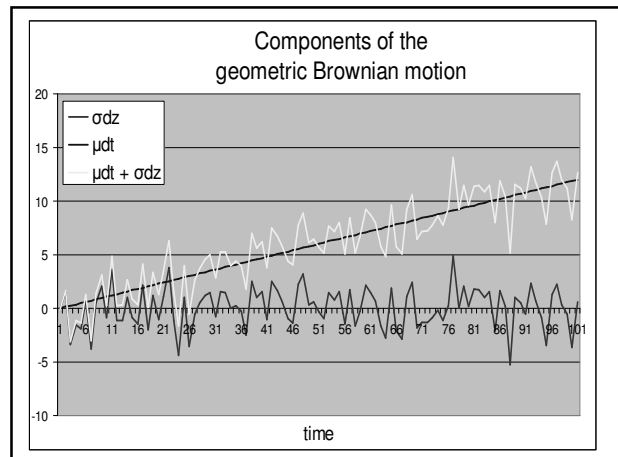
can be rewritten as

$$dS/S = \mu dt + \sigma dz$$

where dS/S can be interpreted as the **return** of the underlying asset.

BLACK-SCHOLES FORMULA

- Two components of the return process:
 1. μdt is the **deterministic trend** component with slope μ and
 2. σdz is the random **noise** component with standard deviation σ .
- Therefore, the return process is simply a noise around a deterministic trend.
- To present this, we graph the return process described by $dS/S = \mu dt + \sigma dz$ as follows:



BLACK-SCHOLES FORMULA

- Notice that:
 - (1) BS assumes a **particular 'real world' probability structure** of the price process S ,
 - (2) the BS model is formulated in **continuous time**, and
 - (3) the BS formula applies only to a limited number of derivatives: European call and put options.

BLACK-SCHOLES FORMULA

- The BS price of a European call option:

$$c = S_0 N(d_1) - X \exp(-rT) N(d_2)$$

$$d_1 = \frac{\ln\left(\frac{S_0}{X}\right) + \left(r + \frac{\sigma^2}{2}\right) T}{\sigma \sqrt{T}}$$

$$d_2 = d_1 - \sigma \sqrt{T}$$

BLACK-SCHOLES FORMULA

Notation:

- S_0 : price of the underlying asset at $t=0$.
- X : strike price of the option.
- r : risk-free rate.
- T : time-to-expiration of the option.
- σ : standard deviation of the return of the underlying asset ("volatility").
- $N(\cdot)$: cumulative distribution function of the standard normal distribution.

BLACK-SCHOLES FORMULA

- The BS price of a European put option:

$$p = X \exp(-rT) N(-d_2) - S_0 N(-d_1)$$

$$d_1 = \frac{\ln\left(\frac{S_0}{X}\right) + \left(r + \frac{\sigma^2}{2}\right) T}{\sigma \sqrt{T}}$$

$$d_2 = d_1 - \sigma \sqrt{T}$$

BLACK-SCHOLES FORMULA

Notation:

- S_0 : price of the underlying asset at $t=0$.
- X : strike price of the option.
- r : risk-free rate.
- T : time-to-expiration of the option.
- σ : standard deviation of the return of the underlying asset ("volatility").
- $N(\cdot)$: cumulative distribution function of the standard normal distribution.

A REMARK ABOUT TIME SCALE

- Notice that the
 - (1) **risk-free rate**, r
 - (2) **time-to-expiration**, T and
 - (3) **volatility**, σare time scale dependent variables.
- In the BS formula, one needs to use **annual** scale for these variables. (annual risk-free rate; T measured in years and standard deviation of annual returns, σ .)

A REMARK ABOUT VOLATILITY

- Notice that in the BS formula one of the parameters is volatility of the return of the underlying asset.
- However, the slope of the deterministic trend, μ is not in the formula.
- The presence of σ in the BS formula means that the 'real world' probability structure of the underlying price process influences the BS option price.
- Remember that in the binomial tree approach it is not like this!!!

VOLATILITY ESTIMATION

Annual volatility estimation

- In the BS formula, we need the volatility of annual returns.
- However, consistent estimation of σ from annual data series is not possible due to small sample size.

Annual volatility estimation

- One solution is to use a higher frequency data, for example daily returns,
 - (1) Estimate volatility of daily returns, then,
 - (2) Rescale daily volatility to annual volatility.
- We can do this in the following way:

Annual volatility estimation

Suppose that:

- (1) We estimate daily volatility, $\sigma_{1\text{day}}$ as follows:

$$\sigma_{1\text{day}} = \sqrt{\frac{1}{n-1} \sum_{t=1}^n (y_t - \bar{y})^2}$$

where y_t is daily return, \bar{y} is the mean of y_t and n is the sample size.

- (2) There are 250 trading days during a year.
- (3) Daily returns are independent random variables.

Annual volatility estimation

- Then, the annual volatility can be rescaled from daily volatility as follows:

$$\sigma_{1\text{year}} = \sigma_{1\text{day}} \sqrt{250}$$

Advantages of this methodology:

- (1) The sample size can be large, therefore the statistical estimation is reliable.
- (2) We use 'relatively recent' return data to estimate annual volatility.

Annual volatility estimation

- **Disadvantage** of this methodology:
- Daily returns are **not** independent random variables!
- There exist more sophisticated dynamic models of volatility showing this fact. (See the GARCH model for example).

COMPUTING THE VALUE OF N(·)

A REMARK ABOUT N(·)

- The cumulative distribution of $N(0,1)$ is given in tables.
- See the table of $N(\cdot)$ left in the copy shop.
- Be familiar about how to obtain a value of $N(x)$ based on that table!
- In Excel, there is a function for the cumulative distribution function of $N(0,1)$:
- **NORMSDIST**(x) - in English Excel
- **DISTR.NORM.ESTAND**(x) - in Spanish Excel

EXERCISE OF N(·)

- See the table for using the table for $x \geq 0$:
 $N(0.6278) =$
 $= N(0.62) + 0.78[N(0.63) - N(0.62)]$
 $= 0.7324 + 0.78x(0.7357 - 0.7324)$
 $= 0.7350$

EXERCISE OF $N(\cdot)$

- See the table for using the table for $x \leq 0$:

$$\begin{aligned} N(-0.1234) &= \\ &= N(-0.12) - 0.34[N(-0.12) - N(-0.13)] \\ &= 0.4522 - 0.34 \times (0.4522 - 0.4483) \\ &= 0.4509 \end{aligned}$$

EXERCISE OF $N(\cdot)$

- Use the table of the $N(0,1)$ distribution to compute $N(x)$ for the next values of x :

- (a) $x = 0.0521$
- (b) $x = 0.1367$
- (c) $x = 2.4701$
- (d) $x = -0.0012$
- (e) $x = -1.5419$
- (f) $x = -2.3177$

(a) $x = 0.0521$

- $N(0.0521) =$
 $N(0.05) + 0.21[N(0.06) - N(0.05)] =$
 $0.5199 + 0.21[0.5239 - 0.5199] =$
0.5207

(b) $x = 0.1367$

- $N(0.1367) =$
 $N(0.13) + 0.67[N(0.14) - N(0.13)] =$
 $0.5517 + 0.67[0.5557 - 0.5517] =$
0.5544

(c) $x = 2.4701$

- $N(2.4701) =$
 $N(2.47) + 0.01[N(2.48) - N(2.47)] =$
 $0.9932 + 0.01[0.9934 - 0.9932] =$
0.9932

(d) $x = -0.0012$

- $N(-0.0012) =$
 $N(-0.00) - 0.12[N(-0.00) - N(-0.01)] =$
 $0.5 - 0.12[0.5 - 0.4960] =$
0.4995

(e) $x = -1.5419$

- $N(-1.5419) =$
 $N(-1.54) - 0.19[N(-1.54) - N(-1.55)] =$
 $0.0618 - 0.19[0.0618 - 0.0606] =$
0.0616

(f) $x = -2.3177$

- $N(-2.3177) =$
 $N(-2.31) - 0.77[N(-2.31) - N(-2.32)] =$
 $0.0104 - 0.77[0.0104 - 0.0102] =$
0.0102

RISK MANAGEMENT OF OPTIONS CONTRACTS

RISK MANAGEMENT OF OPTIONS

- In the previous section, we priced options in a static setup:
- We computed the price of derivatives at time $t = 0$.
- However, in the reality we are in a dynamic setup: $t = 0, \dots, T$.
- This means that investors are interested in the evolution of the prices in their portfolios over time.

RISK MANAGEMENT OF OPTIONS

- In this section, we are going to analyze to risks associated to **European option** contracts in the **BS framework**.
- Remember from the BS formulas that the price of an option is determined by the next five elements:

RISK MANAGEMENT OF OPTIONS

1. Strike price, X
2. Time to expiration, T
3. Risk-free interest rate, r
4. Spot price of the underlying asset, S_0
5. Volatility of the underlying asset return, σ

RISK MANAGEMENT OF OPTIONS

- Notice that in a dynamic setup X and T are fixed in the option contract at time $t = 0$. Thus, they do not change over time.
- However, also notice that r , S_0 and σ change over time. These are the risk factors of option prices.
- (Remark: The S_0 notation may be misleading. It denotes that actual price of the underlying asset. Thus, as time is passing in a dynamic setup, the price of the underlying asset also changes.)

RISK MANAGEMENT OF OPTIONS

- In the remaining part of this section, for simplicity, we are going to assume that r and σ are constant over time.
- We will focus only on the impact of changing S_0 on the option price.

RISK MANAGEMENT OF OPTIONS

- For a portfolio manager, it is important to know the sensitivity of option prices to the price of the underlying asset in order to
- (1) measure the risk of option and
- (2) construct risk-free portfolios of options and underlying assets.

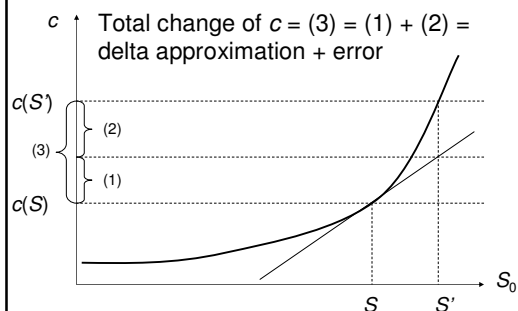
SENSITIVITY OF OPTION PRICE

- We have seen in the BS formulas that the price of an option depends on the price of the underlying asset, S_0 .
- The sensitivity of c and p to S_0 can be approximated by the **partial derivatives** of the $c(S_0)$ and $p(S_0)$ functions with respect to S_0 .

SENSITIVITY OF OPTION PRICE

- Notice that the option price in the BS formulas in a non-linear function of S_0 .
- Therefore, if one considers only the first derivative of $c(S_0)$ and $p(S_0)$ to measure the sensitivity of the option price (i.e., we do a linear approximation), we shall not be precise.
- This fact is presented on the following figure:

APPROXIMATION OF PRICE CHANGE FOR A CALL OPTION



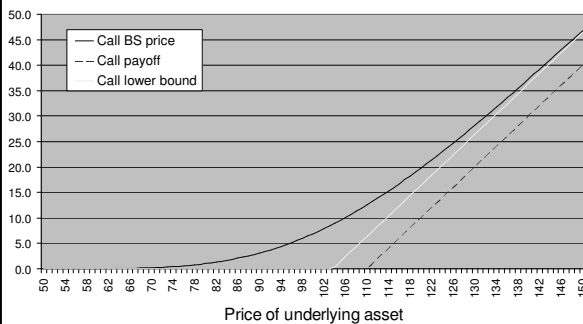
SENSITIVITY OF OPTION PRICE

- We can see on the figure that using the first derivative approximation we conclude that the change of the option price is (1).
- However, the total change of the option price is (3).
- Thus, when we approximate using the first derivative the error we have is (2).
- We can also see that the first derivative approximation is only precise when the change of S_0 is **small**.

OPTION PRICE AS A FUNCTION OF S_0

- On the following slides, we present the non-linearity of call and put option BS prices on figures.
- See the calculation of these figures in the corresponding Excel file.

OPTION PRICE AS A FUNCTION OF S_0 : CALL

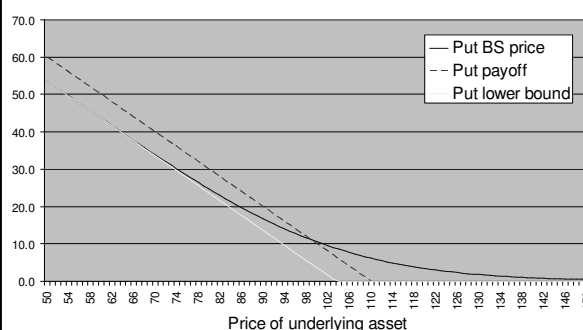


OPTION PRICE AS A FUNCTION OF S_0 : CALL

- Notice that the BS price of the call is always higher than its final payoff.
- In the figure, we also present the lower bound of the call option price, which is given by:

$$\text{Lower bound call}(S_0) = S_0 - X \exp(-rT)$$

OPTION PRICE AS A FUNCTION OF S_0 : PUT



OPTION PRICE AS A FUNCTION OF S_0 : PUT

- Notice that the BS price of the put is lower than its final payoff when the put option is very much in-the-money.
- In the figure, we also present the lower bound of the call option price, which is given by:

$$\text{Lower bound put}(S_0) = X \exp(-rT) - S_0$$

SENSITIVITY OF OPTION PRICE

- In the followings, we shall proceed in two steps:
- First, we shall use the first derivative to measure option price sensitivity.
⇒ DELTA of the option
- Second, we will employ both the first and second derivatives to approximate the sensitivity of option prices.
⇒ DELTA and GAMMA of the option

1. DELTA OF THE OPTION: DELTA HEDGE

1. DELTA

- The first derivative of the option price with respect to the price of the underlying asset is called **delta** and is denoted by Δ .

1. DELTA

- The delta of the call and put options can be approximated as follows in the BS model:

$$\Delta_c = \partial c / \partial S_0 \approx N(d_1) > 0$$

$$\Delta_p = \partial p / \partial S_0 \approx -N(-d_1) = N(d_1) - 1 < 0$$

where d_1 is computed as before:

$$d_1 = \frac{\ln\left(\frac{S_0}{X}\right) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$$

and $N(\cdot)$ is the cumulative distribution function of $N(0,1)$.

1. DELTA

- Notice that on the previous slide we only show an approximate result for the deltas.
- This is because to exact derivative of the $c(S_0)$ and $p(S_0)$ functions are not the reported ones.
- In fact, reviewing the BS formulas, we see that both d_1 and d_2 also depend on S_0 , which should be differentiated too to get the exact formula.

1. DELTA

- However, the presented approximations of delta are precise enough to be used in practice.

1. DELTA

- Notice that the delta of the call option is **positive** while the delta of the put option is **negative**.
- This means that if the price of the underlying product **increases** then the price of the call option will **increase**.
- In addition, if the price of the underlying product **increases** then the price of the put option will **decrease**.

1. DELTA: CALL OPTION SENSITIVITY

- The change of the **call option** price in the BS model can be approximated by delta as follows:

$$dc \simeq \frac{\partial c}{\partial S_0} dS = \Delta dS \simeq [N(d_1)]dS$$

where $dc = c_1 - c_0$ are $dS = S_1 - S_0$ the changes of the option price and underlying asset price between $t = 0$ and $t = 1$, respectively.

1. DELTA: PUT OPTION SENSITIVITY

- The change of the **put option** price in the BS model can be approximated by delta as follows:

$$dp \simeq \frac{\partial p}{\partial S_0} dS = \Delta dS \simeq [N(d_1) - 1]dS$$

where $dp = p_1 - p_0$ are $dS = S_1 - S_0$ the changes of the option price and underlying asset price between $t = 0$ and $t = 1$, respectively.

1. DELTA: SENSITIVITY OF OPTION PRICE

- Why is it useful to compute the **delta** of the option price?
- It is useful because:
 - (1) The $|\Delta|$ can be seen as a **risk measure** of the option price.
 - (2) The $|\Delta|$ defines the so-called **hedge ratio** of the option.

1. DELTA: SENSITIVITY OF OPTION PRICE

- The **hedge ratio** of an option tells us how to construct a **delta neutral** portfolio from
 - (1) the underlying asset and
 - (2) the option.
- When we use the hedge ratio to construct a delta neutral portfolio then we do a so-called **delta hedge**.
- On the next slides, we consider delta hedge for European call and put options.

1. DELTA HEDGE: CALL OPTION

- Consider the following portfolio:
 1. **Buy** $|\Delta|$ units of the underlying asset and
 2. **Sell** 1 unit of a call option on the underlying asset.
- This portfolio is not sensitive to **small** changes of the price of the underlying asset.
- (Remember that the price of the call option is increasing in the price of the underlying asset. See previous figure.)

1. DELTA HEDGE: PUT OPTION

- Consider the following portfolio:
 1. **Buy** $|\Delta|$ units of the underlying asset and
 2. **Buy** 1 unit of a put option on the underlying asset.
- This portfolio is not sensitive to **small** changes of the price of the underlying asset.
- (Remember that the price of the put option is decreasing in the price of the underlying asset. See previous figure.)

1. THE DELTA HEDGE RATIO FORMULA

Disadvantage:

- One can hedge only **small** changes of the underlying asset price.

Advantage:

- The formula includes terms known at time $t = 0$.
- In other words, the risk manager can compute this hedge ratio using information observed at time $t = 0$.

1. DELTA HEDGE

- We shall see an example later for the computation of delta and delta hedge.

2. GAMMA OF THE OPTION: DELTA-GAMMA HEDGE

2. GAMMA: MOTIVATION

- As we have seen before, the price of an option is non-linear function of S_0 in the BS formulas.
- This means that the first derivative is only a linear approximation of the total change of option price for the change of S_0 and works only if the change of S_0 is **small**.

2. GAMMA: MOTIVATION

- A more reliable, non-linear approximation of the sensitivity of option price to S_0 is obtained when we take into account the second partial derivative of the option price as well.
- First, we will present how to compute to second derivative, called **gamma**, of an option in the BS framework.
- Then, we shall approximate the sensitivity of option price using both delta and gamma.

2. GAMMA: DEFINITION

- The second derivative of S_0 is called **gamma** and denoted by Γ .
- In the Black-Scholes model, the **gammas of call and put options are equal** and are computed as follows:

$$\Gamma = \frac{\partial^2 c}{\partial S_0^2} = \frac{\partial^2 p}{\partial S_0^2} = \frac{N'(d_1)}{S_0 \sigma \sqrt{T}}$$

2. GAMMA: DEFINITION

- In the previous formula, $N'(d_1)$ denotes the density function of $N(0,1)$, which can be computed as

$$N'(d_1) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{d_1^2}{2}\right)$$

or in Excel use the following function:

- English: NORMDIST($d_1, 0, 1, \text{FALSE}$)
- Spanish: DISTR.NORM($d_1, 0, 1, \text{FALSO}$)

2. DELTA-GAMMA

- On the following slides, we focus on the delta-gamma approximation of option price sensitivity using the first two terms of the Taylor series of $c(S_0)$ and $p(S_0)$.

2. DELTA-GAMMA: CALL OPTION SENSITIVITY

- Writing the first two terms of the Taylor formula for the $c(S_0)$ function we have:

$$dc \simeq \frac{\partial c}{\partial S_0} dS + \frac{1}{2} \frac{\partial^2 c}{\partial S_0^2} (dS)^2$$

- Substituting the definitions of delta and gamma into this equation we obtain:

$$dc \simeq \Delta dS + \frac{1}{2} \Gamma (dS)^2$$

2. DELTA-GAMMA: CALL OPTION SENSITIVITY

- In the previous formula, dS is the change of the price of the underlying asset between $t = 0$ and $t = 1$:

$$dS = S_1 - S_0$$

- Δ is computed at $t = 0$
- and Γ is computed at $t = 0$
- dc is the approximation of the price change of the call option between $t = 0$ and $t = 1$:

$$dc = c_1 - c_0$$

2. DELTA-GAMMA: PUT OPTION SENSITIVITY

- Writing the first two terms of the Taylor formula for the $p(S_0)$ function we have:

$$dp \simeq \frac{\partial p}{\partial S_0} dS + \frac{1}{2} \frac{\partial^2 p}{\partial S_0^2} (dS)^2$$

- Substituting the definitions of delta and gamma into this equation we obtain:

$$dp \simeq \Delta dS + \frac{1}{2} \Gamma (dS)^2$$

2. DELTA-GAMMA: PUT OPTION SENSITIVITY

- In the previous formula, dS is the change of the price of the underlying asset between $t = 0$ and $t = 1$:

$$dS = S_1 - S_0$$

- Δ is computed at $t = 0$
- and Γ is computed at $t = 0$
- dp is the approximation of the price change of the put option between $t = 0$ and $t = 1$:

$$dp = p_1 - p_0$$

2. DELTA-GAMMA: OPTION SENSITIVITY

- Substituting the computation of delta and gamma for the call and put options priced by the BS formula, we can write the delta-gamma approximation of call and put options as follows:

2. DELTA-GAMMA: OPTION SENSITIVITY

- Call option:

$$dc \simeq N(d_1)dS + \frac{1}{2} \frac{N'(d_1)}{S_0\sigma\sqrt{T}} (dS)^2$$

- Put option:

$$dp \simeq [N(d_1) - 1]dS + \frac{1}{2} \frac{N'(d_1)}{S_0\sigma\sqrt{T}} (dS)^2$$

2. DELTA-GAMMA: SENSITIVITY OF OPTION PRICE

- Why is it useful to compute the **gamma** of the option price?
- It is useful because:
 - (1) The $[\Delta + 0.5 \times \Gamma \times dS]$ value can be used as an alternative risk measure of the option price.
 - (2) The $[\Delta + 0.5 \times \Gamma \times dS]$ value defines an alternative hedge ratio of the option.

2. ALTERNATIVE HEDGE RATIO

- The formula for the alternative hedge ratio is obtained rewriting the equations of the delta-gamma approximation of the call and put options:

$$1dc = \Delta dS + \frac{1}{2}\Gamma(dS)^2 = \left(\Delta + \frac{1}{2}\Gamma dS\right) dS$$

$$1dp = \Delta dS + \frac{1}{2}\Gamma(dS)^2 = \left(\Delta + \frac{1}{2}\Gamma dS\right) dS$$

2. DELTA-GAMMA: SENSITIVITY OF OPTION PRICE

- The hedge ratio of an option tells us how to construct a **delta-gamma neutral** portfolio from
 - (1) the underlying asset and
 - (2) the option.
- When we use this alternative hedge ratio to construct a delta-gamma neutral portfolio then we do a so-called **delta-gamma hedge**.
- On the next slides, we consider delta-gamma hedge for European call and put options.

2. DELTA-GAMMA HEDGE: CALL OPTION

- Consider the following portfolio:
- 1. **Buy** $\Delta + 0.5 \times \Gamma \times dS$ units of the underlying asset and
- 2. **Sell** 1 unit of a call option on the underlying asset.
- This portfolio is not sensitive to **large** changes of the price of the underlying asset.
- (Remember that the price of the call option is increasing in the price of the underlying asset.)

2. DELTA-GAMMA HEDGE: PUT OPTION

- Consider the following portfolio:
- 1. **Buy** $\Delta + 0.5 \times \Gamma \times dS$ units of the underlying asset and
- 2. **Buy** 1 unit of a put option on the underlying asset.
- This portfolio is not sensitive to **large** changes of the price of the underlying asset.
- (Remember that the price of the put option is decreasing in the price of the underlying asset.)

2. THE DELTA-GAMMA HEDGE RATIO FORMULA

Advantage:

- One can hedge **large** changes of the underlying asset price.

Disadvantage:

- The formula includes the $dS=S_1-S_0$ term and S_1 is not known at time $t=0$.
- Thus, the risk manager needs to suppose a future price for the underlying asset.

OPTION RISK MANAGEMENT: TWO EXAMPLES

- You are a financial risk manager of BBVA.
 - You have to
- (Ex.1) Compute the delta and gamma of a **European call** and a **European put** option and approximate the future change of the option prices using these values.
- (Ex.2) Form a delta neutral and a delta-gamma neutral portfolio from the options and their underlying asset.

OPTION RISK MANAGEMENT: TWO EXAMPLES

- On the next slides, we solve these two tasks in examples 1 and 2.

EXAMPLE 1: DELTA-GAMMA APPROXIMATION

- (1) Compute the delta and gamma of a **European call** and a **European put** option and approximate the future change of the option prices using these values.

**EXAMPLE 1:
DELTA-GAMMA APPROXIMATION**

- In part (1a), the delta and gamma values are computed for **European call option** using the BS model.
- In part (1b), the delta and gamma values are computed for **European call put option** using the BS model.

**(1a) DELTA AND GAMMA
COMPUTATION FOR CALL**

- The delta and gamma values are computed for **European call option** using the BS model.
- We shall use the next initial data for both options:

dS	5		$dS=S_T-S_0$
	t=0	t=1	S_0
S	100	105	strike price
X	110	110	risk-free rate
r	3%	3%	expiration date
T	2	2	volatility
σ	15%	15%	

**(1a) DELTA AND GAMMA
COMPUTATION FOR CALL**

- Next, we calculate d_1 , d_2 , $N(d_1)$, $N(d_2)$ and $N'(d_1)$:

	t=0	t=1
d_1	-0.06	0.17
d_2	-0.27	-0.04
$N(d_1)$	0.48	0.57
$N(d_2)$	0.39	0.48
$N'(d_1)$	0.40	0.39

**(1a) DELTA AND GAMMA
COMPUTATION FOR CALL**

- Finally, we compute the values of BS call price, delta, gamma and the corresponding true and approximated changes:

	t=0	t=1	dc	
c	6.92	9.53	2.61050	True change: $dc=c_1-c_0$
Δ	0.4759	0.5673	2.37962	Δ approx.: Δds
Γ	0.0188	0.0177	2.61427	Δ - Γ approx.: $\Delta ds + \frac{1}{2}\Gamma(ds)^2$

**(1a) DELTA AND GAMMA
COMPUTATION FOR CALL**

- **Remark:** When we compute the approximations in the previous table, we should use the values of delta and gamma for $t = 0$ (marked by bold letters).
- In other words, we only use current, known values to approximate the option price change.

**(1b) DELTA AND GAMMA
COMPUTATION FOR PUT**

- The delta and gamma values are computed for **European call put option** using the BS model.
- We shall use the next initial data for both options:

dS	5		$dS=S_T-S_0$
	t=0	t=1	S_0
S	100	105	strike price
X	110	110	risk-free rate
r	3%	3%	expiration date
T	2	2	volatility
σ	15%	15%	

(1b) DELTA AND GAMMA COMPUTATION FOR PUT

- Next, we calculate d_1 , d_2 , $N(d_1)$, $N(d_2)$ and $N'(d_1)$:

	t=0	t=1
d_1	-0.06	0.17
d_2	-0.27	-0.04
$N(-d_1)$	0.52	0.43
$N(-d_2)$	0.61	0.52
$N'(d_1)$	0.40	0.39

(1b) DELTA AND GAMMA COMPUTATION FOR PUT

- Finally, we compute the values of BS call price, delta, gamma and the corresponding true and approximated changes:

	t=0	t=1	dc	
p	10.51	8.12	-2.38950	True change: $dp=p_1-p_0$
Δ	-0.5241	-0.4327	-2.62038	Δ approx.: Δds
Γ	0.0188	0.0177	-2.38573	Δ - Γ approx.: $\Delta ds + \frac{1}{2}\Gamma(ds)^2$

(1b) DELTA AND GAMMA COMPUTATION FOR PUT

- Note: Again, when we compute the approximations in the previous table, we should use the values of delta and gamma for $t = 0$ (marked by bold letters).
- In other words, we only use current, known values to approximate the option price change.

EXAMPLE 2 : DELTA-GAMMA HEDGE

- (2) Form a delta neutral and a delta-gamma neutral portfolio from the options and their underlying asset.

EXAMPLE 2: DELTA-GAMMA HEDGE

- In part (2a), we shall consider the hedge of a **European short call position** by a **long underlying asset position**.
- In part (2b), we will review the hedge of a **European long put position** by a **long underlying asset position**.
- In both parts, we form two alternative portfolios: (*) delta neutral and (**) delta-gamma neutral.
- We use the data from Example 1.

(2a) Hedge of a European short call position by a long underlying asset position

(*) The next table presents a delta neutral portfolio:

asset	units	units	true change
short call	-1	-1	-2.6105
long underlying asset	$ \Delta $	0.4759	2.3796
PORTFOLIO			-0.2309

(2a) Hedge of a European short call position by a long underlying asset position

- (**) The next table presents a delta-gamma neutral portfolio:

asset	units	units	true change
short call	-1	-1	-2.6105
long underlying asset	$ \Delta + 0.5 \times \Gamma \times dS $	0.5229	2.6143
PORTFOLIO			0.0038

(2b) Hedge of a European long put position by a long underlying asset position.

- (*) The next table presents a delta neutral portfolio:

asset	units	units	true change
long put	1	1	-2.3895
long underlying asset	$ \Delta $	0.5241	2.6204
PORTFOLIO			0.2309

(2b) Hedge of a European long put position by a long underlying asset position.

- (**) The next table presents a delta-gamma neutral portfolio:

asset	units	units	true change
long put	1	1	-2.3895
long underlying asset	$ \Delta + 0.5 \times \Gamma \times dS $	0.4771	2.3857
PORTFOLIO			-0.0038

SWAPS

SWAPS

- The swap market is a huge component of the derivatives market.
- Swaps are multi-period extensions of forward contracts.
- These contracts provide a means to quickly, cheaply, and anonymously restructure the balance sheet.
- Therefore, swaps are very frequently used in risk management in practice.

SWAPS

- There are two main types of swap contracts:
 1. **Foreign exchange swap**
 2. **Interest rate swap**

FOREIGN EXCHANGE SWAP

SWAPS - Foreign exchange swap

- Rather than agreeing to exchange two currencies at forward price at one single future date, a **foreign exchange swap (FES)** is an exchange of two currencies at a fixed exchange rate on several future dates.
- This fixed exchange rate is called **swap rate**.
- Thus, FES is a sequence of currency futures contracts.

SWAPS - Foreign exchange swap

Example:

- Two parties might exchange USD 2 million for GBP 1 million in each of the next 5 years.
- In this swap, the USD/GBP exchange rate is fixed for 5 years at 1 GBP = 2 USD that is USD/GBP = 0.5.
- Let $F^* = 0.5$ USD/GBP denote this constant exchange rate called **swap rate**.

SWAPS - Foreign exchange swap

- The investor receiving GBP is going to receive GBP 1 million in each of the next 5 years.
- Similarly, the investor receiving USD is going to receive USD 2 million in each of the next 5 years.

SWAPS - Foreign exchange swap

- We shall investigate whether the $F^* = 0.5$ USD/GBP swap rate in this example is a correct value or not.
- In order to find the fair swap rate, F^* we exploit the analogy between a swap agreement and a sequence of futures contracts.

SWAPS - Foreign exchange swap

- The fair value of F^* in a T -period swap is given by the next formula:

$$\frac{F_1}{(1+y_1)} + \frac{F_2}{(1+y_2)^2} + \dots + \frac{F_T}{(1+y_T)^T} = \frac{F^*}{(1+y_1)} + \frac{F^*}{(1+y_2)^2} + \dots + \frac{F^*}{(1+y_T)^T}$$

where F_t denotes the futures exchange rate for date t and y_t denotes the risk-free rate for date t .

SWAPS - Foreign exchange swap

- To determine the fair value of F^* , the investor needs to substitute the actual futures prices, F_t and the risk-free rates, y_t into the previous equation.
- Then, he needs to solve the equation in order to find F^* .

SWAPS - Foreign exchange swap

- **Data:** To find F^* , we need data on the futures exchange rates for the next 5 years and the risk-free spot yield curve:

t	Futures price USD/GBP	Risk-free spot yield curve
0		
1	0.5	3%
2	0.6	3.20%
3	0.55	3.50%
4	0.6	3.80%
5	0.65	4%

SWAPS - Foreign exchange swap

- Then, we evaluate the previous swap formula for $F^* = 0.5$ USD/GBP:

	DF(t,y)	PV(futures price)	PV(F^*)	squared error
t=0				
t=1	0.971	0.485	0.485	
t=2	0.939	0.563	0.469	
t=3	0.902	0.496	0.451	
t=4	0.861	0.517	0.431	
t=5	0.822	0.534	0.411	
	TOTAL	2.596	2.248	12.14%

SWAPS - Foreign exchange swap

- We can see in the table that the two sides of the equation are not equal and we make a squared error of 12.14%.
- In order to find, the fair swap rate F^* , we need to choose F^* such that the two sides of the equation are equal.
- This we can do using solver in Excel.

SWAPS - Foreign exchange swap

- Employing Solver we get the following result:

t	Futures price USD/GBP	F^*	risk-free spot yield curve	DF(t,y)	PV(futures price)	PV(F^*)	squared error
0							
1	0.5	0.5775	3%	0.971	0.485	0.561	
2	0.6	0.5775	3.20%	0.939	0.563	0.542	
3	0.55	0.5775	3.50%	0.902	0.496	0.521	
4	0.6	0.5775	3.80%	0.861	0.517	0.497	
5	0.65	0.5775	4%	0.822	0.534	0.475	
	TOTAL				2.596	2.596	0.00%

SWAPS - Foreign exchange swap

- **Conclusion:**
- The fair swap rate is given by 0.5775 USD/GBP.

INTEREST RATE SWAP

SWAPS - Interest rate swap

- **Interest rate swaps (IRS)** call for the exchange of a series of cash flows proportional to a fixed interest rate for a corresponding series of cash flows proportional to a floating interest rate.
- The fixed interest rate is called **swap rate**.
- In other words, IRS exchange a fixed interest rate payment for a floating interest rate payment.

SWAPS - Interest rate swap

Example:

- One party might exchange a variable cash flow equal to EUR 1 million times a short-term interest rate (for example EURIBOR) for EUR 1 million times a fixed interest rate of 8% for each of the next 5 years.
- Let $F^* = 8\%$ denote the constant interest rate and call it **swap rate**.

SWAPS - Interest rate swap

- In this swap, the investor receiving the fixed interest rate obtains EUR 1 million x 8% = EUR 80.000 each of the next 5 years.
- Similarly, the investor receiving the floating interest rate obtains EUR 1 million x EURIBOR each of the next 5 years.

SWAPS - Interest rate swap

- We shall investigate whether the $F^* = 8\%$ constant interest rate in this example is a correct value or not.
- In order to find the fair swap rate, F^* , we exploit the analogy between a swap agreement and a sequence of futures contracts.

SWAPS - Interest rate swap

- The fair value of F^* in a T -period swap is given by the next formula:

$$\frac{F_1}{(1+y_1)} + \frac{F_2}{(1+y_2)^2} + \dots + \frac{F_T}{(1+y_T)^T} = \frac{F^*}{(1+y_1)} + \frac{F^*}{(1+y_2)^2} + \dots + \frac{F^*}{(1+y_T)^T}$$

where F_t denotes the EURIBOR futures interest rate for date t and y_t denotes the risk-free rate for date t .

SWAPS - Interest rate swap

- To determine the fair value of F^* , the investor needs to substitute the actual futures prices, F_t and the risk-free rates, y_t into the previous equation.
- Then, he needs to solve the equation in order to find F^* .

SWAPS - Interest rate swap

- **Data:** To find F^* , we need data on the EURIBOR futures rates for the next 5 years and the risk-free spot yield curve:

t	EURIBOR futures rates	risk-free spot yield curve
0		
1	4%	3%
2	4.20%	3.20%
3	4.10%	3.50%
4	4.40%	3.80%
5	4.70%	4%

SWAPS - Interest rate swap

- Then, using the data we evaluate the previous swap formula for $F^* = 8\%$:

	DF(t,y)	PV(futures price)	PV(F^*)	squared error
t=0				
t=1	0.971	0.039	0.078	
t=2	0.939	0.039	0.075	
t=3	0.902	0.037	0.072	
t=4	0.861	0.038	0.069	
t=5	0.822	0.039	0.066	
TOTAL		0.192	0.360	2.82%

SWAPS - Interest rate swap

- We can see in the table that the two sides of the equation are not equal and we make a squared error of 2.82%.
- In order to find, the fair swap rate F^* , we need to choose F^* such that the two sides of the equation are equal.
- This we can do using solver in Excel.

SWAPS - Interest rate swap

- Employing Solver we get the following result:

t	EURIBOR futures rates	F^*	risk-free spot yield curve	DF(t,y)	PV(futures price)	PV(F^*)	squared error
0							
1	4%	4.27%	3%	0.971	0.039	0.041	
2	4.20%	4.27%	3.20%	0.939	0.039	0.040	
3	4.10%	4.27%	3.50%	0.902	0.037	0.038	
4	4.40%	4.27%	3.80%	0.861	0.038	0.037	
5	4.70%	4.27%	4%	0.822	0.039	0.035	
				TOTAL	0.192	0.192	0.00%

SWAPS - Interest rate swap

- **Conclusion:**
- The fair fixed interest rate is given by 4.27%.

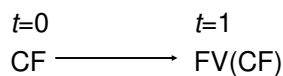
A NOTE ON TRADITIONAL RETURN AND LOG-RETURN

INTRODUCTION

- There exist two alternative ways to account for return / interest:
 1. Traditional return
 2. Log-return or **continuous return**
- When we compute the future value of a cash flow using these ways of accounting returns, we have two alternative formulas.

FUTURE VALUE

- Consider the following cash flow during 1 year:



- For this cash flow, we have two alternative formulas to compute FV:

FUTURE VALUE FORMULAS

- For traditional return:

$$\text{FV}_1(\text{CF}, r, n) = \text{CF} \left(1 + \frac{r}{n}\right)^n$$

where n denotes the number of times interest is paid during one year and r denotes the “traditional” interest rate.

- Note that this formula assumes that we reinvest n times the interest payment.

FUTURE VALUE FORMULAS

- Continuous return:

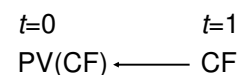
$$\text{FV}_2(\text{CF}, r) = \text{CF} \exp(r)$$

where r denotes the continuous return.

- We will show that this formula implicitly assumes that interest is paid continuously and is reinvested continuously.

PRESENT VALUE

- Consider the following cash flow during 1 year:



- For this cash flow, we have two alternative formulas to compute PV:

PRESENT VALUE FORMULAS

- The present value formulas can be obtained easily for both cases:
- Traditional return:

$$PV_1(CF, r, n) = CF \left(1 + \frac{r}{n}\right)^{-n}$$

- Continuous return:

$$PV_2(CF, r) = CF \exp(-r)$$

RELATIONSHIP BETWEEN THE TWO RETURNS

- The two formulas can be related using the following equation:

$$\lim_{n \rightarrow \infty} \left(1 + \frac{r}{n}\right)^n = \exp(r)$$

- This equation means that if the number of interest payments and interest reinvestments during the year goes to infinity then $FV(CF)$ and $PV(CF)$ are equal.

RELATIONSHIP BETWEEN THE TWO RETURNS

- Thus, we can write the next result:

$$\lim_{n \rightarrow \infty} FV_1(CF, r, n) = FV_2(CF, r)$$

and

$$\lim_{n \rightarrow \infty} PV_1(CF, r, n) = PV_2(CF, r)$$

WHY DO WE USE CONTINUOUS RETURNS?

- In finance, we use continuous returns because it has some mathematical properties that make it easier to use when we do computation or modeling in finance.
- We use it because in many cases it is easier to work with continuous return than with the traditional one.

FINANCIAL STATEMENTS FINANCIAL RATIOS

STURCTURE OF CLASS

1. Major financial statements
2. Ratio analysis

THE MAJOR FINANCIAL STATEMENTS

The major financial statements

Three major financial statements are:

- 1. Income statement**
- 2. Balance sheet**
- 3. Statement of cash flows**

Income statement

Income statement

- The income statement is a summary of the profitability of the firm over a period of time, such as a year.
- It represents revenues generated during the operating period, the expenses incurred during that same period, and the company's net earning or profits, which are simply the difference between revenues and expenditures.

Income statement

- In this class, we shall see relatively simple income statements that contain few accounts.
- In general, an income statement of a large firm may contain thousands of accounts.
- **Example:** The following table presents the income statement of Hewlett-Packard for the year 2009:

INCOME STATEMENT Hewlett-Packard, year 2009

		year 2009
(1)	Net sales	101,658
(2)	Other income	631
(3)=(1+2)	Operating revenues	102,289
(4)	Cost of goods sold	66,825
(5)	Selling, general and administrative expenses	11,266
(6)	Research and development expenses	3,591
(7)	Other expenses	814
(8)=(3-4-5-6-7)	EBITDA	19,793
(9)	Depreciation and amortization	2,353
(10)=(8-9)	EBIT	17,440
(11)	Financial revenues	153
(12)	Financial expenses	249
(13)=(11-12)	Financial result	(96)
(14)	Extraordinary result	(21)
(15)=(10+13+14)	Taxable income	17,323
(16)	Income taxes	993
(17)=(15-16)	Net income	16,330

Income statement

Some abbreviations:

- EBITDA = Earnings before interest, taxes, depreciation and amortization
- EBIT = Earnings before interest and taxes

What is the difference between amortization and depreciation?

- **Amortization** usually refers to spreading an intangible asset's cost over that asset's useful life.
- For example, a patent on a piece of medical equipment usually has a life of 17 years. The cost involved with creating the medical equipment is spread out over the life of the patent, with each portion being recorded as an expense on the company's income statement.

What is the difference between amortization and depreciation?

- **Depreciation**, on the other hand, refers to prorating a tangible asset's cost over that asset's life.
- For example, an office building can be used for a number of years before it becomes run down and is sold. The cost of the building is spread out over the predicted life of the building, with a portion of the cost being expensed each accounting year.

What is the difference between amortization and depreciation?

A remark:

- It is important to note that in some places, such as Canada, the terms amortization and depreciation are often used interchangeably to refer to both tangible and intangible assets.

Balance sheet

BALANCE SHEET

- The balance sheet is the list of the firm's **assets, liabilities** and **shareholders' equity** at a moment.

Balance: ASSETS

- The first section of the balance sheet gives a listing of the assets of the firm.
- **Current assets** are presented first.
- Current assets are cash and other items (accounts receivable, inventories, etc.) that will be converted into cash within 1 year.

Balance: ASSETS

- Next comes the part of **fixed assets**.
- Within fixed assets there is a listing of long-term or **fixed tangible assets**, which usually consists primarily of the company's property, plant, and equipment.
- Another part of fixed assets are **intangible fixed assets** such as goodwill, patents or software.

Balance: What is goodwill?

- Goodwill is an accounting term used to reflect the portion of the book value of a business entity not directly attributable to its assets and liabilities.
- It normally arises only in case of an acquisition.
- It reflects the ability of the entity to make a higher profit than would be derived from selling the tangible assets.
- Goodwill is considered an intangible asset.

Balance: ASSETS

- **Example:** The following table presents the assets in Hewlett-Packard's balance sheet for dates 01/01/2009 and 12/31/2009:

	1/1/2009	12/31/2009
Assets	75,250	77,672
Current assets	45,120	48,264
Cash and marketable securities	15,500	16,400
Receivables	21,000	22,699
Inventories	8,020	7,750
Other current assets	600	1,415
Fixed assets	30,130	29,408
Tangible fixed assets	9,610	9,203
Property, plant and equipment	7,100	6,863
Long-term investments	2,510	2,340
Intangible fixed assets	20,520	20,205
Goodwill	17,400	16,853
Other intangible assets	3,120	3,352

Balance: A remark about long-term investments

- Long-term investments are considered as long-term **financial investments** like bonds, stocks held for more than 1 year.

Balance: LIABILITIES AND EQUITY

- The **liability** and **shareholders' equity** section is arranged similarly to assets:
- First come short-term or **current liabilities** such as accounts payable, accrued taxes, and debts that are due within 1 year.
- Following this is **long-term liabilities** such as debt and other liabilities due in more than 1 year.

Balance: LIABILITIES AND EQUITY

- The difference between total assets and total liabilities is **stockholders' equity**.
- This is the net worth or 'book value' of the firm.
- **Example:** The following table presents the liabilities and equity in Hewlett-Packard's balance sheet for dates 01/01/2009 and 12/31/2009:

	1/1/2009	12/31/2009
Liabilities and Equity	75,250	77,672
Liabilities	49,618	43,837
Current liabilities	40,065	35,850
Debt due for repayment	4,305	2,705
Accounts payable	27,500	25,688
Other current liabilities	8,260	7,457
Long-term liabilities	9,553	7,987
Long-term debt	3,240	2,490
Deferred liabilities	2,650	1,750
Other long-term liabilities	3,663	3,747
Equity	25,632	33,835
Common stock	16,120	17,993
Retained earnings	7,718	9,512
Net income	1,794	6,330

Balance: Deferred liabilities definition

Deferred liabilities:

- Debt where the payment made is postponed beyond the present date.
- An example is deferred taxes.

A remark about retained earnings and net income

- Retained earnings or reserves accumulate the net incomes of past years:
- Retained earnings(t) =
Retained earnings(t-1) + Net income(t-1)

BALANCE SHEET VERSUS INCOME STATEMENT

- While **income statement** provides a measure of profitability **over time**, the **balance sheet** provides a “**snapshot**” of the financial condition of the firm at a particular moment.
- For example, we have HP’s income statement for year 2006 and balance sheets from dates 01/01/2009 and 12/31/2009.

Statement of Cash Flows

STATEMENT OF CASH FLOWS

- The income statement and balance sheets are based on accrual methods of accounting, which means that revenues and expenses are recognized at the time of a sale even if no cash has been exchanged.
- In contrast, the statement of cash flows tracks the cash implications of transactions.

STATEMENT OF CASH FLOWS

- For example, if goods are sold now, with payment due in 60 days, the **income statement** will treat the revenue as generated when the sale occurs, and the **balance sheet** will be immediately augmented by accounts receivable, but the **statement of cash flows** will not show an increase in available cash until the bill is paid.

STATEMENT OF CASH FLOWS

- The statement of cash flows provides evidence on the well-being of a firm.
- If a company cannot pay its dividends and maintain the productivity of its capital stock out of cash from operations, this is a serious warning that the firm cannot maintain the dividend payout at its current level in the long run.
- The statement of cash flows will reveal this developing problem.

STATEMENT OF CASH FLOWS

- Another use of the statement of cash flows is in company valuation.
- In the **discounted cash flow valuation** technique, the researcher gives and estimation of future cash flows available for stockholders and creditors.
- Then, these cash flow forecasts are discounted at appropriate interest rates in order to determine the company's present value.

STATEMENT OF CASH FLOWS

- In this class, we shall study the construction of statements of cash flows that can be used for company valuation purposes.
- We remark that the accounting law of every country defines the components and the structure of the statement of cash flows.

STATEMENT OF CASH FLOWS

- Although there is a international convergence of national accounting laws, the financial statements are different among countries and more importantly they are created not only for valuation purposes.

STATEMENT OF CASH FLOWS

- In order to get the cash flow statement for a given period (for example year 2009), one needs the following **three** accounting documents:
- Income statement of the period (year 2009)
- Balance sheet of the first day of the period (date 01/01/2009)
- Balance sheet of the last day of the period (date 12/31/2009)

STATEMENT OF CASH FLOWS

- The reason why we need these three documents is that the values presented in the **income statement** are **flow type values** that can included directly in the statement of cash flows.
- However, the values in the **balance** are snapshots taken on specific dates (i.e., **stock type values**).

STATEMENT OF CASH FLOWS

- In order, to get **flow type values** presented in the **cash flow statement** from the balance sheet one needs the compute the difference between the values of last and the first days of the period.

STATEMENT OF CASH FLOWS

- The general structure of the cash flow statement is presented in the next table:

(1)	Cash provided by operations
(2)	Cash flows from investments
(3)	Extraordinary cash flow
(1+2+3)	FCFF = Free cash flow to firm
(4)	Cash flow from financing activities
(1+2+3+4)	FCFE = Free cash flow to equity

(1) Cash provided by operations

- This is the cash flow directly coming from the firm's operations.
- For example, in the case of Volkswagen car manufacturer company, the cash provided by operations is the cash flow related to car production, salary payments, and the sale of Volkswagen Polo cars.
- This cash flow under normal circumstances has positive value.

(2) Cash flows from investments

- This is the cash flow related to the firm's investments in fixed non-financial assets.
- For example, in case of Volkswagen, this is the cash flow related to purchase of machines related to car production, plants, properties, and other tangible assets.
- This cash flow under normal circumstances has negative value.

(3) Extraordinary cash flow

- This cash flow is related to extraordinary events occurred during the year.
- For example, these extraordinary events can be labor accidents, natural disasters or other events that are not directly related to the normal operations of the firm.

FCFF = Free cash flow to firm

- The sum of (1), (2) and (3) is called **free cash flow to firm (FCFF)**.
- FCFF is an important cash flow category because it represents the cash available to:
 - (1) **Shareholders** who finance the company by purchasing equity (stocks) and
 - (2) **Creditors** who finance the firm through liabilities (short- and long-term debts).
- FCFF should be positive normally.

(4) Cash flow from financing activities

- This is the cash flow related to financial assets and liabilities and financial results of the company.
- The cash flow from financing activities reflect the changes of the value of financial assets and liabilities and the profit or loss of financial operations.
- Under normal conditions, the cash flow from financing activities is negative due to the interest payments to creditors.

FCFE = Free cash flow to equity

- The sum of FCFF and (4) Cash flow from financing activities is called **free cash flow to equity** (FCFE).
- FCFE is also an important cash flow category because it represents the cash available to:
 - (1) **Shareholders** who finance the company by purchasing equity (stocks).
- The FCFE normally should be positive.

MORE DETAILED CASH FLOW STATEMENT

More detailed structure of the Cash flow statement

- The first row in the cash flow statement is always EBIT.
- In the second row, we subtract the tax payment to compute after-tax EBIT.
- The tax payment can be defined in various ways and as a consequence we have different cash flow statements depending on how we compute the tax payment.

More detailed structure of the Cash flow statement

- On the following slides, two alternative cash flow statements are presented:
 - CF1: Cash flow where Income taxes are subtracted from EBIT**
 - CF2: Cash flow where Tax due to operating result is subtracted from EBIT**
- We do this to compute the **Net Operating Profit Less Adjusted Taxes (NOPLAT)**.

More detailed structure of the Cash flow statement

- The difference between these two alternatives is that CF1 and CF2 are two alternatives to compute tax.
- In CF1, all the income tax presented in the income statement is subtracted from EBIT.
- In CF2, only a proportion of the income tax is subtracted from EBIT. This portion is the tax that corresponds to the operating activity and it is called 'Tax due to operating result'.

More detailed structure of the Cash flow statement

- The CF2 can be seen as a better cash flow because in CF1 we not only subtract the tax corresponding to the operating activity from EBIT but also subtract the taxes that correspond to the extraordinary events and financial operations that have nothing to do with the basic operations of the firm.

More detailed structure of the Cash flow statement

● Therefore, when we do CF2, we decompose to tax payment into the following parts:

- (1) Tax due to operating result
- (2) Tax due to extraordinary result
- (3) Tax due to financial result

More detailed structure of the Cash flow statement

Computation tax payments in CF2:

$$\text{Tax due to operating result} = \text{EBIT} \times \frac{\text{Income taxes}}{\text{Taxable income}}$$

$$\text{Tax due to extraordinary result} = \text{Extraordinary result} \times \frac{\text{Income taxes}}{\text{Taxable income}}$$

$$\text{Tax due to financial result} = \text{Financial result} \times \frac{\text{Income taxes}}{\text{Taxable income}}$$

More detailed structure of the Cash flow statement

- On the following slides, both CF1 and CF2 cash flow statements are presented.
- Afterwards, some remarks about the construction of the cash flow statements will be presented.

	year 2009	Origin of account
CF1		
EBIT	17,440	Income statement
- Income taxes	993	Income statement
NOPLAT	16,447	
+ Depreciation and amortization	2,353	Income statement
- Δ Current assets (without cash!)	2,244	Balance sheet
+ Δ Current liabilities	(4,215)	Balance sheet
Cash provided by operations	12,341	
- Δ Property, plant and equipment	(237)	Balance sheet
- Δ Intangible fixed assets	(315)	Balance sheet
- Depreciation and amortization	2,353	Income statement
Cash flows from investments	(1,801)	
Extraordinary result	(21)	Income statement
Extraordinary cash flow	(21)	
FCFF	-10,519	
- Δ Long-term investments	(170)	Balance sheet
+ Δ Long-term liabilities	(1,566)	Balance sheet
+ Financial revenues	153	Income statement
- Financial expenses	249	Income statement
Cash flow from financing activities	(1,492)	
FCFE	-9,027	
+ Δ Equity	8,203	Balance sheet
- Net income	16,330	Income statement
Δ Cash and marketable securities	900	
Δ Cash and marketable securities	900	Balance sheet

	year 2009	Origin of account
CF2		
EBIT	17,440	Income statement
- Tax due to operating result	1,000	Income statement
NOPLAT	16,440	
+ Depreciation and amortization	2,353	Income statement
- Δ Current assets (without cash!)	2,244	Balance sheet
+ Δ Current liabilities	(4,215)	Balance sheet
Cash provided by operations	12,334	
- Δ Property, plant and equipment	(237)	Balance sheet
- Δ Intangible fixed assets	(315)	Balance sheet
- Depreciation and amortization	2,353	Income statement
Cash flows from investments	(1,801)	
+ Extraordinary result	(21)	Income statement
- Tax due to extraordinary result	(1)	Income statement
Extraordinary cash flow	(20)	
FCFF	-10,519	
- Δ Long-term investments	(170)	Balance sheet
+ Δ Long-term liabilities	(1,566)	Balance sheet
+ Financial revenues	153	Income statement
- Financial expenses	249	Income statement
- Tax due to financial result	(6)	Income statement
Cash flow from financing activities	(1,486)	
FCFE	-9,027	
+ Δ Equity	8,203	Balance sheet
- Net income	16,330	Income statement
Cash and marketable securities	900	
Cash and marketable securities	900	Balance sheet

REMARKS

1. Notice that all accounts in the cash flow, which come from the balance sheet start with the Δ notation.

- The Δ means the we include the change of the value of the account in the cash flow.
- For example:

$$\Delta \text{ Current liabilities} = \text{Current liabilities}(t) - \text{Current liabilities}(t-1)$$

REMARKS

2. Notice that when we compute Δ Current assets, we do not take into account the **Cash and marketable securities account** from the balance sheet.

- This means that

$$\Delta \text{ Current assets} = \Delta \text{ Receivables} + \Delta \text{ Inventories} + \Delta \text{ Other current assets}$$

- This is because **Δ Cash and marketable securities** are presented in the last row of the cash flow table.

REMARKS

3. Related to remark 2, notice that the last two rows of the cash flow statement are **Δ Cash and marketable securities** two times.

- The first row is a result of the previous rows of the table (see the computation in the first column).
- The second row directly comes from the balance sheet by Δ Cash and marketable securities = Cash and marketable securities(t) - Cash and marketable securities(t-1).
- These two rows provide a verification of the cash flow calculation. If everything has been done well, these two rows should be equal.

REMARKS

4. Notice that the cost of **depreciation and amortization** is removed from **cash provided by operations** and included in **cash flows from investments**.

- This is due to the fact that depreciation is a cost related to the firm's investments in fixed assets and not to its operations.

REMARKS

5. Notice that an **increase** in the value of any the **assets**, i.e.

$$\Delta \text{ Asset} = \text{Asset}(t) - \text{Asset}(t-1) > 0$$

always has **negative** impact on the cash flow. (See the first column of the cash flow tables for this.)

- This is because more assets mean more investments of the firms.

REMARKS

6. Notice that an **increase** in the value of any the **liabilities or equity**, i.e.

$$\Delta \text{ Liability} = \text{Liability}(t) - \text{Liability}(t-1) > 0$$

or

$$\Delta \text{ Equity} = \text{Equity}(t) - \text{Equity}(t-1) > 0$$

always has **positive** impact on the cash flow. (See the first column of the cash flow tables for this.)

- This is because more equity/liability means more financing of the firm.

REMARKS

7. Notice that **revenues** have **positive** while **costs** have **negative** impact on the firm's cash flow. (See the first column of the cash flow tables for this.)

RATIO ANALYSIS

INTRODUCTION

- The major financial statements give an important financial and economic information of a firm's situation.
- However, these documents contain a large amount of information that is difficult to overview.
- Therefore, in the practice analysts and company managers use several financial ratios to summarize the main accounting documents of the company.

INTRODUCTION

- These financial ratios are typically used in order to compare
 1. Different firms at a given point of time. (For example, compare the profitability of two competitor firms in 2009 using a profitability ratio.)
 2. Different years of the same firm. (For example, to study the evolution of the profitability of Hewlett-Packard during 2000-2009.)

INTRODUCTION

Remark:

- When the following financial ratios include one item from the **income statement**, which covers a period of time, and another from the **balance sheet**, which is a "snapshot" at a particular time, the practice is **to take the average of the beginning and the end-of-year balance sheet figures**.

INTRODUCTION

- We shall classify the financial ratios in the next groups:
 1. **Leverage ratios**
 2. **Asset utilization ratios**
 3. **Liquidity ratios**
 4. **Profitability ratios**
 5. **Market price ratios**

LEVERAGE RATIOS

LEVERAGE RATIOS

- The following ratios will be presented:

1. **Interest burden**
2. **Interest coverage**
3. **Leverage**
4. **Compound leverage factor**

1. Interest burden

$$\text{Interest burden} = \frac{\text{EBIT} - \text{Interest expense}}{\text{EBIT}}$$

- We shall explain this ratio later.

2. Interest coverage

$$\text{Interest coverage} = \frac{\text{EBIT}}{\text{Interest expense}}$$

- A high coverage ratio indicates that the likelihood of bankruptcy is low because annual earnings are significantly greater than annual interest obligations.
- It is widely used by both lenders and borrowers in determining the firm's debt capacity and is a major determinant of the firm's **bond rating**.

3. Leverage

$$\text{Leverage} = \frac{\text{Assets}}{\text{Equity}} = 1 + \frac{\text{Debt}}{\text{Equity}}$$

- We shall explain this ratio later.

4. Compound leverage factor

$$\text{Compound leverage factor} = \text{Interest burden} \times \text{Leverage}$$

- We shall explain this ratio later.

ASSET UTILIZATION RATIOS

ASSET UTILIZATION RATIOS

- The following ratios will be presented:

1. **Total asset turnover**
2. **Fixed asset turnover**
3. **Inventory turnover**
4. **Days receivables**

1. Total asset turnover

$$\text{Total asset turnover} = \frac{\text{Sales}}{\text{Average total assets}}$$

- This ratio measures sales per EUR of the firm's money tied up in **assets**.

2. Fixed asset turnover

$$\text{Fixed asset turnover} = \frac{\text{Sales}}{\text{Average fixed assets}}$$

- This ratio measures sales per EUR of the firm's money tied up in **fixed assets**.

3. Inventory turnover

$$\text{Inventory turnover} = \frac{\text{Cost of goods sold}}{\text{Average inventories}}$$

- The numerator of this ratio is the cost of goods sold instead of sales revenue because inventory is valued at cost.
- This ratio measures the speed with which inventory is turned over.

4. Days receivables

$$\text{Days receivables} = \frac{\text{Average accounts receivable}}{\text{Annual sales}} \times 365$$

- This ratio is also called **average collection period**.
- It is the number of days' worth of sales tied up in accounts receivable.
- An alternative interpretation of this ratio:
- It is the average lag between the date of sale and the date payment is received.

LIQUIDITY RATIOS

LIQUIDITY RATIOS

- The following ratios will be presented:

1. **Current ratio**
2. **Quick ratio**
3. **Cash ratio**

1. Current ratio

$$\text{Current ratio} = \frac{\text{Current assets}}{\text{Current liabilities}}$$

- This ratio measures the ability of the firm to pay off its current liabilities by **liquidating its current assets**.

2. Quick ratio

$$\text{Quick ratio} = \frac{\text{Cash} + \text{Marketable securities} + \text{Receivables}}{\text{Current liabilities}}$$

- This ratio is also called the **acid test ratio**.
- The quick ratio is a better measure of liquidity than the current ratio for firms whose **inventory is not readily convertible into cash**.

3. Cash ratio

$$\text{Cash ratio} = \frac{\text{Cash} + \text{Marketable securities}}{\text{Current liabilities}}$$

- A company's receivables are also less liquid than its holdings of cash and marketable securities.
- Therefore, the cash ratio is a better measure than quick ratio for firms whose receivable **may not be obtained in the future**.

PROFITABILITY RATIOS

PROFITABILITY RATIOS

- The following ratios will be presented:
1. **Return on assets (ROA)**
 2. **Return on equity (ROE)**
 3. **Return on sales (ROS) = Profit margin**

1. Return on assets (ROA)

$$\text{Return on assets} = \frac{\text{EBIT}}{\text{Average total assets}}$$

2. Return on equity (ROE)

$$\text{Return on equity} = \frac{\text{Net income}}{\text{Average stockholders' equity}}$$

3. Return on sales (ROS) (= Profit margin)

$$\text{Return on sales (Profit margin)} = \frac{\text{EBIT}}{\text{Sales}}$$

ROE versus ROA

- ROE and ROA can be related by the following formula:

$$\text{ROE} = (1 - \text{Tax rate}) \left[\text{ROA} + (\text{ROA} - \text{Interest rate}) \frac{\text{Debt}}{\text{Equity}} \right]$$

- On the following slides, this formula is demonstrated:

ROE versus ROA

- Proof:

$$\begin{aligned} \text{ROE} &= \frac{\text{Net income}}{\text{Equity}} = \\ &= \frac{\text{EBIT} - \text{Interest payment} - \text{Income taxes}}{\text{Equity}} = \\ &= \frac{(1 - \text{Tax rate})(\text{EBIT} - \text{Interest payment})}{\text{Equity}} = \end{aligned}$$

ROE versus ROA

- Proof (*continued*):

$$\begin{aligned} &= \frac{1 - \text{Tax rate}}{\text{Equity}} (\text{ROA} \times \text{Assets} - \text{Interest rate} \times \text{Debt}) = \\ &= \frac{1 - \text{Tax rate}}{\text{Equity}} [\text{ROA} \times (\text{Equity} + \text{Debt}) - \text{Interest rate} \times \text{Debt}] = \\ &= (1 - \text{Tax rate}) \left[\text{ROA} + (\text{ROA} - \text{Interest rate}) \frac{\text{Debt}}{\text{Equity}} \right] \end{aligned}$$

Implications of the previous ROE vs ROA relationship

If there is no debt or the ROA equals the interest rate on the firm's debt:

- ROE will simply equal $(1 - \text{Tax rate}) \times \text{ROA}$.

Implications of the previous ROE vs ROA relationship

If there is debt:

- If ROA exceeds the interest rate then its ROE **will exceed** $(1 - \text{Tax rate}) \times \text{ROA}$ by an amount that will be greater the higher the debt-to-equity ratio.
- If the interest rate exceeds ROA then its ROE **will be lower** than $(1 - \text{Tax rate}) \times \text{ROA}$ by an amount that will be greater the higher the debt-to-equity ratio.

Implications of the previous ROE vs ROA relationship

These results make intuitive sense:

- If ROA exceeds the interest rate, the firm earns more than it pays out to creditors. The surplus earnings are available to the firm's equityholders, which increases ROE.
- If, on the other hand, ROA is less than the interest rate then ROE will decline by an amount that depends on the debt-to-equity ratio.

DuPont system

- To understand the factors affecting a firm's ROE, particularly its trend over time and its performance relative to competitors, analysts often decompose ROE into the product of a series of ratios.
- Each component ratio is itself meaningful, and the process serves to focus the analyst's attention on the separate factors influencing performance.

DuPont system

- This kind of decomposition is often called the **DuPont system**.
- (The name comes from the name of the chemical company, which has made it popular.)
- One useful decomposition to 5 factors is presented on the following slide:

ROE =

$$\frac{\text{Net income}}{\text{Taxable income}} \times \quad (\text{Factor 1})$$

$$\frac{\text{Taxable income}}{\text{EBIT}} \times \quad (\text{Factor 2})$$

$$\frac{\text{EBIT}}{\text{Sales}} \times \quad (\text{Factor 3})$$

$$\frac{\text{Sales}}{\text{Assets}} \times \quad (\text{Factor 4})$$

$$\frac{\text{Assets}}{\text{Equity}} \quad (\text{Factor 5})$$

Factor 1

$$\text{Factor 1} = \frac{\text{Net income}}{\text{Taxable income}}$$

- Factor 1 is called the **tax burden ratio**.
- Its value reflects both the government's tax code and the policies pursued by the firm in trying to minimize its tax burden.

Factor 2

$$\text{Factor 2} = \frac{\text{Taxable income}}{\text{EBIT}} = \frac{\text{EBIT} - \text{Interest expense}}{\text{EBIT}}$$

- Factor 2 is called **interest-burden ratio**.
- The firm's pretax profits will be the greatest when there are no interest payments to be made to debtholders.
- Its highest possible value is 1.

Factor 3

$$\text{Factor 3} = \frac{\text{EBIT}}{\text{Sales}}$$

- Factor 3 is known as the firm's operating **profit margin** or **return on sales (ROS)**.

Factor 4

$$\text{Factor 4} = \frac{\text{Sales}}{\text{Assets}}$$

- Factor 4, the ratio of sales to total assets, is known as **total asset turnover (ATO)**.
- It indicates the efficiency of the firm's use of assets in the sense that it measures the annual sales generated by each EUR of assets.

Factor 5

$$\text{Factor 5} = \frac{\text{Assets}}{\text{Equity}} = 1 + \frac{\text{Debt}}{\text{Equity}}$$

- Factor 5 is called the **leverage ratio**.
- It is a '*direct*' measure of the firm's financial leverage.
- However, in the previous discussion about the relationship between ROE and ROA, we have seen that leverage helps boost ROE only if ROA is greater than the interest rate.

Factor 5

- How is this '*indirect*' fact reflected in the DuPont system?
- To measure the full impact of leverage the analyst must take the product of the **interest burden** and **leverage** ratios (i.e., Factors 2 and 5).
- This product is called **compound leverage factor**:

$$\text{Compound leverage factor} = \text{Interest burden} \times \text{Leverage}$$

DuPont system

- We can summarize the DuPont system decomposition of ROE using the previous ratio definitions as follows:

ROE =

Tax burden × (Factor 1)

Interest burden × (Factor 2)

Profit margin × (Factor 3)

Total asset turnover × (Factor 4)

Leverage (Factor 5)

DuPont system

- An alternative decomposition can be obtained by considering the facts that:

**ROA = Profit margin X Total asset turnover
= Factor 3 X Factor 4**

and

**Compound leverage factor =
Interest burden X Leverage =
Factor 2 X Factor 5**

DuPont system

- Substituting these into the previous ROE decomposition, we get the following alternative of the ROE decomposition:

ROE =

Tax burden X

ROA X

Compound leverage factor

MARKET PRICE RATIOS

MARKET PRICE RATIOS

- The following ratios will be presented:
 1. Market-to-book (P/B)
 2. Price-earnings ratio (P/E)
 3. Earnings yield

1. Market-to-book (P/B)

$$\text{Market-to-book} = \frac{\text{Price per share}}{\text{Book value per share}}$$

- The market-book-value ratio (P/B) equals the market price of a share of the firm's common stock divided by its book value.
- Remark: Book value is shareholders' equity in the Balance sheet.

1. Market-to-book (P/B)

- Analysts sometimes consider the stock of a firm with a low P/B value to be a "safer" investment, seeing the book value as the "floor" supporting the market price.

1. Market-to-book (P/B)

- Analysts presumably view book value as the level below which market price will not fall because the firm always has the option to liquidate, or sell, its assets for their book value.
- However, this view is questionable.
- Nevertheless, low P/B ratio is seen by some as providing a "margin of safety," and some analysts will screen out or reject high P/B ratio firms in their stock selection process.

1. Market-to-book (P/B)

- An alternative interpretation of the P/B ratio is as a **measure of growth opportunities**.
- Empirically, firms with greater growth opportunities tend to exhibit higher multiples of P/B.

2. Price-earnings ratio (P/E)

$$\text{Price-earnings ratio} = \frac{\text{Price per share}}{\text{Earnings per share}}$$

- Remark: Earnings = Net income
- While low P/E stocks allow you to pay less per EUR of current earnings, high P/E stocks may still be better bargain if its earnings are expected to grow quickly.
- Nevertheless, most analysts believe that low P/E stocks are more attractive than high P/E stocks.

3. Earnings yield (E/P)

$$\text{Earnings yield} = \frac{\text{Earnings per share}}{\text{Price per share}}$$

- Remark: Earnings = Net income

Relationship among ROE, P/B and P/E ratios

- We have the following relationship among ROE, P/B and P/E:

$$\text{ROE} = \frac{\text{Earnings}}{\text{Book value}} =$$

$$\frac{\text{Market price}}{\text{Book value}} \div \frac{\text{Market price}}{\text{Earnings}} =$$

$$\text{P/B} \div \text{P/E}$$

Relationship among ROE, P/B and E/P ratios

- Rearranging the previous equation we get that earnings yield can be expressed by ROE divided by the market-book-value ratio:

$$\frac{E}{P} = \frac{\text{ROE}}{\text{P/B}}$$

- Thus, a company with high ROE can have relatively low earnings yield because its P/B ratio is high.

COMPANY VALUATION *DISCOUNTED CASH FLOW (DCF) AND OTHER METHODS*

INTRODUCTION

- The objective of this class is to present some commonly applied techniques of firm valuation.
- After the introduction, we present four valuation methods:

INTRODUCTION

1. **DCF valuation**
 2. **Economic value added (EVA) model**
 3. **Relative valuation – multiples**
 4. **Contingent claim valuation**
- We will talk more about the DCF method.
 - Valuation methods will be presented in a general setup. Therefore, they can be applied for several enterprises in many industries.

INTRODUCTION - What do we value?

- See the next simplified balance sheet:

Assets	Equity
	Liabilities (Debt)

INTRODUCTION - What do we value?

- In the balance sheet, the total value of assets is equal to the sum of equity and liabilities.
- Therefore, we may value either
 - (1) **Equity** or
 - (2) **Assets = Equity + Debt**

INTRODUCTION - What do we value?

- The **stockholders** of the firm are interested in maximizing the value of the **equity**.
- In other words, investors who buy the company's stocks on the financial market are interested in the value of equity.

INTRODUCTION - What do we value?

- The **creditors**, for example banks, who provide external financing to the company are interested in both the value of the **debt** and the total value of **assets**.
- For example, many credits are secured by specific assets of the firm (collateral). In this case, the bank is interested in the value of these assets.

FIRM VALUATION METHODS

FIRM VALUATION METHODS

- We review four approaches to firm valuation:
 1. **DCF valuation** ("mark-to-model")
 2. **Economic value added (EVA)** model
 3. **Relative valuation – multiples** ("mark-to-market")
 4. **Contingent claim valuation** (using option pricing models, firms are "real options")

1. DCF valuation - STRUCTURE

- a) Introduction
- b) Cost of capital calculation
- c) Analyze historical performance
- d) Forecasting cash flow
- e) Present value of free cash flow

1. DCF valuation - Introduction

1. DCF valuation - Introduction

- This method is based on computing the present value of future CFs generated by the company.
- When we use this method we need to analyze historical performance, value drivers and predict future CFs.
- This CF can be either
 - (1) CF to firm (FCFF) or
 - (2) CF to equity (FCFE).

1. DCF valuation - Introduction

- The estimation of CFs using the balance sheets and income statement, we have already seen in the Financial Statements section.
- In order to compute the value of firm or equity, we have to discount FCFF and FCFE, respectively.
- To do this, we need a discount rate (cost of capital), which is different for FCFF and FCFE.

1. DCF valuation - Introduction

- The DCF method can be applied easily when:
 1. We value firms with currently positive CF that can be estimated with reliability for future periods.
 2. A proxy for risk that can be used to obtain discount rates is available.

1. DCF valuation - Introduction

- The further we get from this idealized setting, the more difficult DCF valuation becomes.
- Some examples when DCF valuation may be **problematic**:

1. DCF valuation - Introduction

(a) Firms in trouble:

- A distressed firm generally has negative earnings and CFs currently and expects to lose money for some time in the future.
- For these firms estimating future CFs is difficult to do, since there is a strong probability of bankruptcy.
- For firms which are expected to fail, DCF valuation does not work very well since it looks at firms as providing positive CFs.

1. DCF valuation - Introduction

(b) Cyclical firms:

- The earnings and CFs of cyclical firms tend to follow the economy – rising during economic booms and falling during recessions.
- If DCF valuation is used for these firms, expected future CFs are usually smoothed out, unless the analyst undertakes the task of predicting the time and duration of economic recessions and recoveries.

1. DCF valuation - Introduction

(c) Firms with unutilized assets:

- DCF valuation reflects the value of all assets that produce CFs.
- If a firm has assets that are unutilized (i.e., do not produce any CFs), the value of these assets will not be reflected in DCF valuation.
- Nevertheless, the value of these assets can be obtained externally and then added to the value obtained by the DCF method.

1. DCF valuation - Introduction

(d) Firms with patents or product options:

- Firms often have unutilized patents or product options that do not produce any current CFs and are not expected to produce CFs in the near future, but nevertheless are valuable.
- If this is the case, the DCF value will understate the true value of the firm.
- Again we can overcome this problem by valuing externally then adding to the DCF.

1. DCF valuation - Introduction

(e) Firms in the process of restructuring:

- Firms in the process of restructuring often sell some of their assets, acquire other assets, and change their capital structure and dividend policy. Some of them also change their ownership structure.
- Each of these changes makes the estimation of future CFs difficult.
- However, if future CFs and discount rates reflect the effects of these changes, DCF may be useful.

1. DCF valuation - Introduction

(f) Firms involved in acquisitions:

- There are at least two specific issues to be taken into account when using DCF method to value target firms:
 1. Is there synergy in the merger and its value can be estimated?
 2. (Especially in hostile takeovers.) What is the effect of changing management on CFs and discount rates?

1. DCF valuation - Introduction

(g) Private firms:

- The biggest problem of using DCF method to value private firms is estimating the discount rates, since most risk-return models require that parameters be estimated from historical prices on the asset being analyzed. Since securities on private firms are not traded this is not possible.
- A solution is to look at the risk of comparable firms publicly traded.

1. DCF valuation – Cost of capital

1. DCF valuation – Cost of capital

- In the financial statements section, we have already seen how to get different kinds of CFs from the income statement and two consecutive balance sheets.
- However, we also need a discount factor to compute the present value of CF.
- For different types of CF different interest rates are needed for the discount factor.

1. DCF valuation – Cost of capital

1. The **FCFE** is discounted using the **cost of capital of equity** (also called expected return on equity), r_E
2. The **FCFF** is discounted using the **Weighted Average Cost of Capital (WACC) after taxes**.

1. DCF valuation – Cost of capital COST OF CAPITAL OF EQUITY

- The **cost of capital of equity** can be estimated in several ways:
1. **Capital asset pricing model (CAPM)**
 2. **Arbitrage pricing theory (APT)**
 3. **Dividend growth model**

1. DCF valuation – Cost of capital 1. CAPM

- The CAPM determines the expected return of equity as follows:

$$r_E = r_f + \beta_E (r_M - r_f)$$

where

r_f is the risk-free rate (for example the yield to maturity of Treasury bills or Treasury bonds) and

r_M is the expected return of the stock market (for example the return of the market index like S&P500 or IBEX).

1. DCF valuation – Cost of capital 1. CAPM

- $(r_M - r_f)$ is the **market risk premium**,
- β_E (called “beta”) is a parameter that captures the risk associated to the return of equity of firm E , and
- r_E : is the expected return of equity of firm E .

1. DCF valuation – Cost of capital 1. CAPM

- As β_E is different for each company, it can be considered as a risk measure of the firm’s equity.
 - β_E measures how the stock market return affects the stock return of firm E .
 - β_E can be either positive or negative:
1. If the market index increases and $\beta_E > 0$ then the stock price increases.
 2. If the market index increases and $\beta_E < 0$ then the stock price decreases.

1. DCF valuation – Cost of capital 2. APT

- Another popular model of expected stock return is the APT.
 - It is formulated as follows:
- $$r_E = c + \beta_{E1} F_1 + \dots + \beta_{EN} F_N$$
- where F_i denotes the i -th risk factor of the stock return.
 - APT can be seen as an extension of CAPM because CAPM is a one-factor model where the factor is the market risk premium.

**1. DCF valuation – Cost of capital
Estimation of CAPM and APT**

● The parameters of CAPM and APT can be estimated by ordinary least squares (OLS) using the following linear regressions:

● CAPM:

$$r_{Et} = r_{ft} + \beta_E (r_{Mt} - r_{ft}) + \varepsilon_{Et}$$

● APT:

$$r_{Et} = c + \beta_{E1} F_{1t} + \dots + \beta_{EN} F_{Nt} + \varepsilon_{Et}$$

● where the variables are observed over $t=1, \dots, T$ and ε_{Et} is the i.i.d error term with $E[\varepsilon_{Et}] = 0$ and $\text{Var}[\varepsilon_{Et}] = \sigma^2$.

**1. DCF valuation – Cost of capital
3. Dividend growth model**

● This approach is based on the DCF model that can be used to value a firm in stable growth.

● For a firm, which has a stable growth rate in earnings and dividends, the present value of CFs on a share of equity can be written as follows:

**1. DCF valuation – Cost of capital
3. Dividend growth model**

$$S_0 = \frac{DIV_1}{r_E - g}$$

where S_0 is the price of a stock today, DIV_1 are dividends per share next year, r_E is the expected yield on equity, and g is the growth rate of dividends.

● Notice that this formula is the present value of a growing perpetuity.

**1. DCF valuation – Cost of capital
3. Dividend growth model**

● Manipulating this formula we get:

$$r_E = \frac{DIV_1}{S_0} + g$$

● This formula can be used to estimate the expected return of equity.

● The formula tells us that r_E equals to the sum of **expected dividend yield** and **growth rate in dividends**.

**1. DCF valuation – Cost of capital
3. Dividend growth model**

Advantage:

● This approach is simple.

Disadvantage:

● It is limited to steady state firms. It is sensitive to estimates to growth in dividends.

**1. DCF valuation – Cost of capital
COST OF CAPITAL OF FIRM**

● The cost of capital of firm is the weighted average of the costs of the different components of financing.

● We shall focus of two components of financing: (1) equity and (2) debt.

● Define the **WACC after taxes** as follows:

**1. DCF valuation – Cost of capital
WACC after taxes**

$$\text{WACC after taxes} = r_E \frac{E}{E + D} + r_D(1 - T_c) \frac{D}{E + D}$$

where

r_D = pretax expected return of debt

$r_D(1 - T_c)$ = after-tax expected return of debt

T_c = corporate tax rate

r_E = expected return of equity

D = **market value** of debt

E = **market value** of equity

**1. DCF valuation – Cost of capital
WACC: Cost of debt**

- The value of the firm's debt equals the present value of the CF to the debt holders, discounted at a rate that reflects the riskiness of that CF.
- The discount rate should equal the current market rate on similar-risk debt with comparable terms.
- The **after-tax expected return of debt**, $r_D(1 - T_c)$ measures the current cost to the firm of borrowing funds to finance projects.

**1. DCF valuation – Cost of capital
WACC: Cost of debt**

- Cost of debt is determined by:
 - (1) The current level of interest rates
 - (2) The default risk of the company (Use the bond ratings of the firm. Higher ratings lead to lower interest rates.)
 - (3) The tax advantage associated with debt (Since interest is tax deductible, the after-tax cost of debt is a function of the tax rate. This benefit increases as the tax rate T_c increases.)

**1. DCF valuation – Cost of capital
WACC: Why market value is used?**

- The cost of capital measures the cost of issuing securities and these securities are issued at **market value**, not at **book value**.
- Some say that book value is more reliable because it is more stable than market value, i.e. it is not so volatile.
- However, this is more a weakness of book value measure because the true value of firms changes as firm-specific and market-wide information is revealed.

**1. DCF valuation – Cost of capital
Example 1**

- The annual yield to maturity of Spanish treasury bonds is 3%. The annual return of IBEX is 15%. The "beta" of the firm is 2.
- Use the CAPM to compute the expected return of equity of firm E !

**1. DCF valuation – Cost of capital
Example 1**

- **Solution:**

$$\begin{aligned} r_E &= r_f + \beta (r_M - r_f) = \\ &= 3\% + 2(15\% - 3\%) = \\ &= 3\% + 2 \times 12\% = \\ &= 3\% + 24\% = \\ &= 27\% \end{aligned}$$

1. DCF valuation – Cost of capital Example 2

Using the expected return of equity obtained in example 1, compute the WACC after taxes knowing that:

- The yield of the debt is 6%.
- The corporate tax rate is 15%.
- The market value of debt of firm *E* is 10 Millions EUR.
- The stock market capitalization of firm *E* is 20 Millions EUR.

1. DCF valuation – Cost of capital Example 2

● Solution:

WACC after taxes =

$$r_D(1-T_c)D/(D+E) + r_E E/(D+E) =$$

$$6\%(1-15\%)10m/(10m+20m) +$$

$$27\% \times 20m/(10m+20m) =$$

$$6\% \times 85\% \times 10m/30m +$$

$$27\% \times 20m/30m =$$

$$1,7\% + 18\% =$$

$$\mathbf{19,7\%}$$

1. DCF valuation – Analyze historical performance

1. DCF valuation - Analyze historical performance

- The first step in valuing a business is analyzing its historical performance.
- A sound understanding of the company's past cash flows provides an essential perspective for developing and evaluating forecasts of future cash flows.
- Historical performance analysis should focus on the key drivers of cash flow. (See the value drivers section later in the EVA section.)

1. DCF valuation - Analyze historical performance

- The historical analysis should be developed in an integrated way.
- For example, in addition to return on invested capital (ROIC) itself, the key components of ROIC should be analyzed.
- The historical analysis should not be just a collection of ratios. Financial ratios that largely duplicate the core ratios or ratios that do not help to explain the core drivers of cash flow should not be used.

1. DCF valuation - Analyze historical performance

- Once we have calculated the determinants of historical cash flow, we analyze the results looking for trends and making comparisons with other companies in the same industry.
- We can assemble this analysis into an integrated perspective, which combines the financial analysis with an analysis of the industry structure and qualitative assessment of the company's strengths and vulnerabilities.

1. DCF valuation - Analyze historical performance

- Developing this integrate perspective is not a mechanical process, so it is difficult to generalize.
- While it is impossible to provide a comprehensive checklist for understanding a company's historical performance, here are some things to keep in mind:

1. DCF valuation - Analyze historical performance

- (1) Look back in time as far as possible (at least ten years). This will help to understand whether the company and the industry tend to revert to some normal level of performance over time and whether short term trends are likely to be permanent breaks from the past.

1. DCF valuation - Analyze historical performance

- (2) Go as deep into the value drivers as you can, getting as close to operational performance measures as possible.
- (3) If there are any radical changes in performance, identify the source of the change and determine whether it is real or perhaps just an accounting effect and whether any change is likely to be sustained.

1. DCF valuation - Analyze historical performance

- The final step in the historical analysis is understanding the financial health of the company from a credit perspective.
- Here we are not concerned with value creation itself, but with how the company has been financing its value creation.
- To explain the company's financial health from a credit perspective we need to answer to the following questions:

1. DCF valuation - Analyze historical performance

- Is the company generating or consuming cash?
- How much debt does the company employ relative to equity?
- What margin of safety does the company have with respect to its debt financing?

1. DCF valuation – Forecasting

1. DCF valuation – Forecasting

- Once we have analyzed the company's historical performance, we can move on to forecasting its future performance.
- While we cannot provide specific forecasting rules, we can consider the following basic steps when developing forecasts:

1. DCF valuation – Forecasting

- (1) Evaluate the company's strategic position, considering both the industry characteristics as well as the company's competitive advantages or disadvantages.
- This will help to assess the company's growth potential and its ability to earn returns above its cost of capital.

1. DCF valuation – Forecasting

- (2) Develop performance scenarios for the company and its industry that describe qualitatively how the company's performance will evolve and the critical events that are likely to impact that performance.

1. DCF valuation – Forecasting

- (3) Forecast individual income-statement and balance-sheet line items based on the scenarios.
- These line items will then be aggregated to forecast free cash flow.
- (4) Check the overall forecast for reasonableness, particularly the key determinants of value (i.e., value drivers).

1. DCF valuation – Forecasting

- On the following slides some details of the previous points are presented.

1. DCF valuation – Forecasting

(1) Evaluate strategic position

**1. DCF valuation – Forecasting
Evaluate strategic position**

- In order to earn returns on capital in excess of the opportunity cost of capital, companies must develop and exploit a competitive advantage.
- Without a competitive advantage, competition would force all the companies in the industry to earn only their cost of capital.

**1. DCF valuation – Forecasting
Evaluate strategic position**

- Therefore, to develop a point of view about a company's ability to earn an attractive profit on capital invested, we must identify the company's potential for generating competitive advantage, given the nature of the industry in which it competes and its own assets and capabilities.

**1. DCF valuation – Forecasting
Evaluate strategic position**

- Competitive advantages that influence positively the value of the company can be categorized into three types:
- (1) Providing superior value to the customer through a combination of price and product attributes that cannot be replicated by competitors.
- These attributes can be tangible ("the fastest computer") or intangible ("a strong relationship between salesperson and buyer").

**1. DCF valuation – Forecasting
Evaluate strategic position**

- (2) Achieving lower cost than competitors.
- (3) Utilizing capital more productively than competitors.
- A competitive advantage must ultimately be expressed in terms of one or more of these characteristics.
- Describing competitive advantages this way also helps to begin to shape the financial forecast.

**1. DCF valuation – Forecasting
Evaluate strategic position**

- Two techniques for identifying competitive advantages are:
- (1) Customer segmentation analysis and
- (2) Industry structure analysis

- On the following slides we review these techniques.

**1. DCF valuation – Forecasting
Customer segmentation analysis**

- The purpose of customer segmentation analysis is to help estimate potential market share by explicitly identifying why customers will choose one company's products over others.
- It also tells us how difficult it will be for a competitor to differentiate itself and helps to identify how profitable each type of customer is likely to be.

**1. DCF valuation – Forecasting
Customer segmentation analysis**

- Customer segmentation analysis segments customers from two perspectives (1) the customer and (2) the producer.

**1. DCF valuation – Forecasting
Customer segmentation analysis**

- From the customer perspective, product attributes have different importance to different groups of customers.
- For example, after-sale service may be more important to a small manufacturing customer than a large customer with its own in-house maintenance staff.
- In addition, different competitors may include different attributes in their product offering and deliver different benefits to customer groups.

**1. DCF valuation – Forecasting
Customer segmentation analysis**

- A customer segment is a group of customers to whom similar product attributes provide similar benefits.
- Segmenting customers forces the analyst to understand why competitive market share may differ across customer groups and to find opportunities for differentiation to segments.

**1. DCF valuation – Forecasting
Customer segmentation analysis**

- For example, in the overnight package delivery business, detailed billing information on each package is important to some customers, while others are content with summary data.
- Some customers want to be able to know instantaneously where a package is at any point in time, while other customers can wait for the information.

**1. DCF valuation – Forecasting
Customer segmentation analysis**

- From the producer perspective, different customers have different costs to serve.
- For example, in the salt industry, distance to the customer has a major impact on the cost on the cost to serve because of salt's low value-to-weight characteristics.
- Accordingly, some customers may be simply too far away to serve if competitors are much closer.

**1. DCF valuation – Forecasting
Customer segmentation analysis**

- By segmenting customers according to both the customer and producer attributes, and then comparing a company's ability to satisfy those customers relative to competitors, we can begin to identify current or potential competitive advantages.

1. DCF valuation – Forecasting Industry structure analysis

- In company valuation, it is useful to analyze competitors of the firm that we want to value.
- A useful methodology of industry analysis is the **Porter five forces model**.
- It was suggested by Michael Porter in 1979.
- The Porter model has the following five elements:

1. DCF valuation – Forecasting Industry structure analysis

PORTER'S FIVE FORCES:

1. The threat of **substitute products** or services
2. The threat of the **entry of new competitors**
3. The **intensity of competitive rivalry**
4. The **bargaining power of customers** (buyers)
5. The **bargaining power of suppliers**

1. DCF valuation – Forecasting Industry structure analysis

1. The threat of substitute products or services

- A market is not attractive for a company when substitute products or services exist outside the market.
- The situation is even more complicated when these substitute products are technologically more advanced or have lower prices than actual products on the market.

1. DCF valuation – Forecasting Industry structure analysis

2. The threat of the entry of new competitors

- Profitable markets that yield high returns, will draw firms.
- This results in many new entrants, which eventually will decrease profitability.
- Unless the entry of new firms can be blocked by incumbents, the profit rate will fall towards a competitive level (perfect competition).

1. DCF valuation – Forecasting Industry structure analysis

3. The intensity of competitive rivalry

- For most industries, the intensity of competitive rivalry is the major determinant of the competitiveness of the industry.
- It is more difficult to compete on a market where there are many competitors that create entry barriers and fixed costs are high.

1. DCF valuation – Forecasting Industry structure analysis

3. The intensity of competitive rivalry

- If intensity of competition is high then the firm will face price wars, aggressive advertising campaigns of competitors or entrance of new products.

1. DCF valuation – Forecasting Industry structure analysis

4. The bargaining power of customers (buyers)

- The bargaining power of customers is also described as the market of outputs: the ability of customers to put the firm under pressure, which also affects the customer's sensitivity to price changes.
- Well-organized buyers can ask for price reductions, higher quality of products and services decreasing the company's profit margin.

1. DCF valuation – Forecasting Industry structure analysis

5. The bargaining power of suppliers

- The bargaining power of suppliers is also described as the market of inputs.
- Suppliers of raw materials, components, labor, and services (such as expertise) to the firm can be a source of power over the firm, when there are few substitutes.
- Suppliers may refuse to work with the firm, or, e.g., charge excessively high prices for unique resources.

1. DCF valuation – Forecasting Industry structure analysis

Summary:

- A detailed analysis of these five forces can give the analyst a complex picture of the firm's industry.
- It can help to identify the value drivers that affect future CFs and cost of capital of the firm.
- Therefore, it can help in CF prediction and present value calculation as well.

1. DCF valuation – Forecasting

(2) Develop performance scenarios

1. DCF valuation – Forecasting Develop performance scenarios

- Once we have developed a point of view about the company's ability to achieve and sustain a competitive advantage, the next step is to develop performance scenarios.
- By using scenarios we acknowledge that forecasting financial performance is at best an educated guess.
- The best we can do is narrow down the range of likely future performance.

1. DCF valuation – Forecasting Develop performance scenarios

- **Example:** Consider a high tech company that is developing a new product.
- If the company successfully develops the product, its competitive advantage will be a product that delivers superior value to customers.
- Accordingly, its growth and returns on capital are likely to be huge.
- If it fails to develop the product, it will likely go out of business.

**1. DCF valuation – Forecasting
Develop performance scenarios**

- A scenario that projects moderate growth and returns is highly unlikely, even though it could be considered “most likely” from a statistical perspective.
- It is better to develop a number of scenarios for a company, and to understand the company’s value under each scenario, than to build a single “most likely” forecast and value.

**1. DCF valuation – Forecasting
Develop performance scenarios**

- Once the scenarios are developed and valued, an overall value of the company can be estimated as a weighted average of the values of the independent scenarios, assigning probabilities to each scenario.

1. DCF valuation – Forecasting

(3) Forecasting individual line items

**1. DCF valuation – Forecasting
Forecasting individual line items**

- Before forecasting individual line items we must decide on the structure of the forecast.
- The structure of the forecast is the order in which they relate to each other.
- The best forecast structure begins with an integrated income statement and balance sheet forecast.
- The free cash flow can then be derived from them.

**1. DCF valuation – Forecasting
Forecasting individual line items**

- It is possible to forecast free cash flow directly rather than going through the income statement and balance sheet.
- However, it is not recommended to forecast this way.
- If we forecast free cash flow first, we must still construct the balance sheet to properly evaluate the relationships between the cash flow or income statement items and the balance sheet accounts.

**1. DCF valuation – Forecasting
Forecasting individual line items**

- If we do not construct the balance sheet, it is easy to lose sight of how all the pieces fit together.
- The balance sheet also helps to identify the financing implications of the forecast:
- It shows how much capital must be raised or how much excess cash will be available.

**1. DCF valuation – Forecasting
Forecasting individual line items**

- Forecasts of individual line items should draw upon a careful analysis of industry structure and a company's internal capabilities.
- Analyzing the historical levels of valuation variables is a useful starting point.
- Once these levels have been calculated, several questions will help gain insights into the future level of each valuation variable:

**1. DCF valuation – Forecasting
Forecasting individual line items**

- (a) What characteristics of the industry have had the greatest impact on value drivers in the past?
- (b) What company-specific capabilities have had the greatest impact on historical value drivers?

**1. DCF valuation – Forecasting
Forecasting individual line items**

- (c) Are industry characteristics and company capabilities expected to maintain historical patterns in the future? If not, what is expected to change?
- (d) What must change in the industry or the company to cause a significant shift in the historical level of the company's value drivers?

1. DCF valuation – Forecasting

(4) Check overall forecast for reasonableness

**1. DCF valuation – Forecasting
Check overall forecast for reasonableness**

- The final step in the forecasting process is to construct the free cash flow and value drivers from the income statements and balance sheets and to evaluate the forecast.
- To understand how cash flow and value drives are expected to behave, we may ask the following questions:

**1. DCF valuation – Forecasting
Check overall forecast for reasonableness**

- (1) Is the company's performance on the key value drivers consistent with the company's economics and the industry competitive analysis?
- (2) Is revenue growth consistent with industry growth?

1. DCF valuation – Forecasting
Check overall forecast for reasonableness

- (3) If the company's revenue is growing faster than the industry's, which competitors are losing share? Will they retaliate?
- (4) Does the company have the resources to manage that rate of growth?
- (5) Is the return on capital consistent with the industry's competitive structure?
- (6) If entry barriers are coming down, shouldn't expected returns decline?

1. DCF valuation – Forecasting
Check overall forecast for reasonableness

- (7) If customers are becoming more powerful, will margins decline?
- (8) Conversely, if the company's position in the industry is becoming much stronger, should one expect increasing returns?
- (9) How will returns and growth look relative to the competition?
- (10) How will technology changes affect returns? Will they affect risk?

1. DCF valuation – Forecasting
Check overall forecast for reasonableness

- (11) Can the company manage all the investment it is undertaking?
- (12) Will the company have to raise large amounts of capital? If so, can it obtain the financing? Should it be debt or equity?
- (13) If the company is generating excess cash, what options does it have for investing the cash or returning it to shareholders?

Three additional topics

1. DCF valuation – Forecasting
Three additional topics of forecasts

- On the following slides we mention three additional topics related to forecasting free cash flow:
 - (1) **Inflation**,
 - (2) **Length of the forecast**, and
 - (3) **Extraordinary items**

1. DCF valuation – Forecasting
Inflation

- It is recommended that financial forecasts and discount rates be estimated in nominal rather than real currency units.
- For consistency, both the free cash flow forecast and the discount rate must be based on the same expected general inflation rate.

1. DCF valuation – Forecasting Inflation

- Individual line items, however, could have specific inflation rates that are higher or lower than the general rate, but they should still drive off the general rate.
- We can derive the expected general inflation rate that is consistent with the discount rate from the term structure of interest rates.
- The term structure is based on yields to maturity of government bonds.

1. DCF valuation – Forecasting Inflation

- The term structure of interest rates provides a market-based estimate of expected inflation over time.
- This should be the best estimate for valuation purposes for three reasons:
- (1) Most economics', or econometric, forecasts of inflation rarely extend beyond one or two years, far too short a period for valuation.

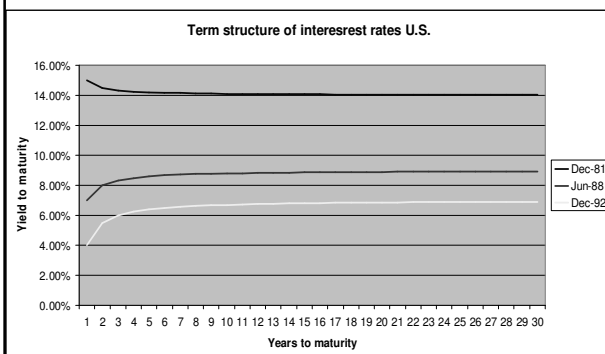
1. DCF valuation – Forecasting Inflation

- (2) Market-based estimates provide a broader consensus view (with investors' money at stake) than individual forecasts.
- (3) Empirical analysis suggests that market-based estimates are the least biased.

1. DCF valuation – Forecasting Inflation

- The next figure shows an example of the term structure of interest rates at different points of time in the U.S.:

1. DCF valuation – Forecasting Inflation



1. DCF valuation – Forecasting Inflation

- On the previous figure we can see that in December 1981 the market expected that future inflation will be lower than near-term inflation.
- On the other hand, in June 1988 and December 1992 the market expected that future inflation will be higher than near-term inflation.

1. DCF valuation – Forecasting Inflation

- The expected inflation rate over a given period can be derived from the term structure of interest rates as follows:
- (1) Determine the nominal risk-free rate of interest for the time period over which we want to estimate inflation.

1. DCF valuation – Forecasting Inflation

- (2) Estimate the real rate of interest.
- This is a controversial issue and different researchers suggest different ways to do it:
 - For example, Copeland et al (2000) suggests using the long-term growth rate in GDP, which is 2-3 percent for the U.S. economy.

1. DCF valuation – Forecasting Inflation

- (3) Use the Fisher equation that relates nominal interest rate and real interest rate as follows:

$$(1 + \text{nominal rate}) = (1 + \text{expected inflation}) \times (1 + \text{real rate})$$

- From this equation we can express the inflation rate as follows:

$$\text{Expected inflation} = \frac{(1 + \text{Nominal rate})}{(1 + \text{Real rate})} - 1$$

1. DCF valuation – Forecasting Length of forecast

- For practical purposes, most forecasts should be divided into two periods:
- (1) an **explicit forecast period** (say, ten years) and
 - (2) the remaining life of the company (year 11 on) = **continuing value period**.

1. DCF valuation – Forecasting Length of forecast

- A detailed forecast is done for the first period.
- Free cash flow from the second, more distant, period is valued using the perpetuity valuation formula.
- The explicit forecast period should be long enough so that the company reaches a steady state by the end of the period.

1. DCF valuation – Forecasting Length of forecast

- The steady state can be described as follows:
- (a) The company earns a constant rate of return on all new capital invested during the continuing value period.
 - (b) The company earns a constant rate of return on its base level of invested capital.
 - (c) The company invests a constant proportion of its earnings back into the business each year.

1. DCF valuation – Forecasting Length of forecast

- The most common approach is to make the forecast period as long as you expect the company to have sustainable rates of return on new investment above the company's cost of capital.
- Microeconomic analysis suggests that over time, competition will drive the returns in many industries to the level of the cost of capital.

1. DCF valuation – Forecasting Length of forecast

- Once the company's returns have converged on its cost of capital, it is relatively simple to estimate the company's continuing value.
- Therefore, forecasting until convergence simplifies the continuing value problem.
- If we use this approach, the forecast periods should be for as long as returns above the WACC are sustainable.

1. DCF valuation – Forecasting Length of forecast

- When in doubt, make a longer rather than shorter forecast.
- We would rarely use a forecast period of less than seven years.
- The forecast period should never be determined by the company's own internal planning period.
- Just because the company forecasts out only three years does not justify using three years for valuation.

1. DCF valuation – Forecasting Length of forecast

- If a company is in a cyclical industry, it is important that your forecast capture complete cycle.
- Failure to do so may result in wildly unrealistic continuing value assumptions, because the up or down phase of the cycle may be projected to last forever.
- It is best to put long-run forecasts (averaging out cyclical effects) into the continuing value assumptions.

1. DCF valuation – Forecasting Extraordinary items

- Financial documents always contain some extraordinary events that are partly reflected in the extraordinary result of the income statement.
- The extraordinary elements of financial documents are recommended to be excluded from all financial documents before forecasting free cash flow.

1. DCF valuation – Present value of free cash flow

1. DCF valuation – Present value of free cash flow

- After forecasting the free cash flow, we compute the present value of FCFE and FCFF to get the value of the firm.
- The present value of FCFE is the **value of equity**.
- The present value of FCFF is the **value of equity + value of debt** (i.e., the value of total assets).

1. DCF valuation – Present value of free cash flow

- The FCFE values have to be discounted using the **expected return on equity**, r_E .
- The FCFF values must be discounted using the **Weighted Average Cost of Capital (WACC) after taxes**.

1. DCF valuation – Present value of free cash flow

- For the first, explicit forecast period, we can use the next formulas in order to calculate the present value of free cash flows:

1. DCF valuation – Present value of free cash flow

- Computation of present value of a cash flow obtained at year t , CF_t :
$$PV(CF_t) = CF_t / (1+r)^t$$
- Present value of FCFE:
$$PV(FCFE_t) = FCFE_t / (1+r_E)^t$$
- Present value of FCFF:
$$PV(FCFF_t) = FCFF_t / (1+WACC)^t$$

1. DCF valuation – Present value of free cash flow

- For the second, continuing value period we use the following formulas to value the cash flow of a growing perpetuity:

1. DCF valuation – Present value of free cash flow

- Suppose that in steady state the cash flow is a growing perpetuity:
$$t=0, \quad t=1, \quad t=2, \quad t=3, \dots$$
$$0 \quad CF_1 \quad CF_2 \quad CF_3$$
where the growth rate of the cash flow is g :
$$CF_{t+1} = (1+g) \times CF_t$$

1. DCF valuation – Present value of free cash flow

- The present value of a growing perpetuity at time $t=0$ is given by:

$$PV(\text{growing perpetuity with rate } g) = \frac{CF_1}{(r-g)}$$

where

CF_1 = cash flow at $t=1$

r = expected return

g = growth rate

1. DCF valuation – Present value of free cash flow

- Assuming that the continuing value period starts at year $t=6$, we have the next growing perpetuity:

$t=0$	$t=1$	$t=2$	$t=3$	$t=4$	$t=5$	$t=6$	$t=7$	$t=8$
0	0	0	0	0	0	CF_6	$CF_6(1+g)$	$CF_6(1+g)^2$

- Therefore, the present value of the free cash flow over the continuing value period is:

$$PV(\text{CF continuing value}) = \frac{CF_6}{[(r-g)(1+r)^5]}$$

1. DCF valuation – Present value of free cash flow

Example: Cash flow forecasts for the following 6 years (2009-2014):

	2009	2010	2011	2012	2013	2014
FCFF	75.7	89.2	105.0	123.7	145.7	171.6
FCFE	73.1	88.6	107.5	130.3	158.0	191.6

Suppose that the expected returns and perpetuity growth rate are:

WACC	10%
r_E	15%
g	3%

1. DCF valuation – Present value of free cash flow

- The FCFE and FCFF values for 2009-2013 are discounted using the following formulas:

$$PV(\text{FCFE}_t) = \frac{\text{FCFE}_t}{(1+r_E)^t}$$

$$PV(\text{FCFF}_t) = \frac{\text{FCFF}_t}{(1+WACC)^t}$$

- The cash flow for the continuing value period is discounted as follows:

$$PV(\text{continuing value}) = \frac{CF_6}{[(r-g)(1+r)^5]}$$

1. DCF valuation – Present value of free cash flow

In the next table, we present the present value of FCFE and FCFF:

	2009	2010	2011	2012	2013	Continuing value
PV(FCFF)	68.8	73.7	78.9	84.5	90.5	1522.1
PV(FCFE)	63.5	67.0	70.7	74.5	78.6	793.9
PV(TOTAL: FCFF)	1918.5					
PV(TOTAL: FCFE)	1148.2					

1. DCF valuation – Present value of free cash flow

- The total present value is computed using the next formulas:

$$PV(\text{TOTAL: FCFE}) = \left[\sum_{t=1}^5 \frac{\text{FCFE}_t}{(1+r_E)^t} \right] + \frac{\text{FCFE}_6}{[(r_E - g)(1+r_E)^5]}$$

$$PV(\text{TOTAL: FCFF}) = \left[\sum_{t=1}^5 \frac{\text{FCFF}_t}{(1+WACC)^t} \right] + \frac{\text{FCFF}_6}{[(WACC - g)(1+WACC)^5]}$$

1. DCF valuation – Present value of free cash flow

Solution:

- The value of equity of the firm is **1148.2 EUR**.
- The value of total assets is **1918.5 EUR**.

2. Economic value added (EVA)

2. EVA – Structure

- In this section, first we will introduce the concept of value drivers.
- Then, we define EVA and present the firm valuation model using EVA.
- Finally, we show that the value provided with DCF coincides with EVA valuation in our setup.

2. EVA – Value drivers

- To understand the idea of valuation by EVA we need to identify the underlying economic value drivers of the business.
- Since value is based on discounted cash flow, the underlying value drivers of the business must also be the drivers of free CF.

2. EVA – Value drivers

- There are two key drivers of free CF and, as a consequence, value:
 1. The **net investment rate** that the company invests from NOPLAT in each period to earn additional profits and
 2. The **return on (operating) invested capital (ROIC)**
- These value drivers are consistent with common sense:

2. EVA – Value drivers

- A company that invests more (i.e., grows faster) will worth more than a company that invests less (i.e., grows slower) if they are both earning the same return on invested capital.
- Similarly, a company that earns higher profits for every dollar invested in the business will worth more than a similar company that earns less profits for every dollar of invested capital.

2. EVA – Value drivers ROIC

- We shall demonstrate how **investment rate** and **ROIC** actually drive CF.
- First, some definitions are needed:

$$\text{ROIC} = \frac{\text{NOPLAT}}{\text{Operating invested capital}}$$

where NOPLAT = Net operating profits less adjusted taxes. (We have seen this in the Financial statements section.)

2. EVA – Value drivers Operating invested capital

- What is operating invested capital?
- **Operating invested capital =**
Operating working capital +
Net property, plant and equipment +
Net other operating assets

2. EVA – Value drivers Operating working capital

- **Operating working capital =**
 - (1) Operating current assets –
 - (2) Noninterest bearing current liabilities
- (1) **Operating current assets** = all current assets used in or necessary for the operations of the business
- (2) **Noninterest bearing current liabilities** = (accounts payable + accrued expenses + other noninterest bearing liabilities)

2. EVA – Value drivers Other net operating fixed assets

- **Net property, plant and equipment** = The book value of the company's property, plant and equipment related to the operations of the business net of noncurrent, noninterest bearing liabilities.
- **Net other operating assets** = other operating assets, net of other liabilities. Any other assets net of noninterest bearing liabilities that are related to the operations of the business.

2. EVA – Value drivers Simplifying assumptions

- We present a simple example to show how the two value drivers impact CF and firm value.

Assumptions:

- Assume that the firm lives infinite periods.
- Assume that in the initial year NOPLAT = 100 Euros.
- Assume that **net investment rate** and **ROIC** are constant.

2. EVA – Value drivers Simple CF statement

- We will work with the following simplified FCFF statement:

	NOPLAT
	- Net investment (Δ operating invested capital)
	= FCFF

- Remark: **net investment** is the change of the value of **operating invested capital**.
- First, we compute FCFF for several periods and show that **the FCFF growth rate is the product of investment rate and ROIC**.

2. EVA – Value drivers

- See the “Value drivers.xls” file for the demonstration of this fact for our setup.
- Remarks about computation:
 1. Net investment(t) =
= NOPLAT(t) x Net investment rate
 2. FCFF(t) = NOPLAT(t) – Net investment(t)
 3. NOPLAT(t+1) =
= NOPLAT(t)+[Net investment(t) x ROIC]

2. EVA – Value drivers

- In the xls table, you can see that the growth rate of FCFF is given by:

$$\text{Growth rate} = \text{Net investment rate} \times \text{ROIC}$$

- Next, we compute the present value of future FCFF to obtain the value of the firm, i.e. we use the DCF valuation approach.

2. EVA – Value drivers

- Let WACC denote the cost of capital of the firm. We shall use WACC to discount future FCFF.
- To compute the present value of CF, we use the PV formula for growing perpetuity:
Value = FCFF₁ / (WACC – g)
where FCFF₁ is FCFF in the first year and g is the FCFF growth rate.

2. EVA – Value drivers

- In the xls file, we compute firm value for alternative values of Net investment rate, ROIC and WACC:

Scenario	ROIC	WACC	Net investment rate	g	Value	Remark
1a	20%	12%	25%	5%	1071	ROIC>WACC
1b	20%	12%	40%	8%	1500	ROIC>WACC
2a	12%	20%	42%	5%	388	ROIC<WACC
2b	12%	25%	67%	8%	195	ROIC<WACC
3a	20%	20%	25%	5%	500	ROIC=WACC
3b	20%	20%	40%	8%	500	ROIC=WACC

2. EVA – Value drivers

- In the table, we can see that:
 1. If ROIC > WACC
then Net investment will **increase** value.
 2. If ROIC < WACC
then Net investment will **decrease** value.
 3. If ROIC = WACC
then Net investment will **not change** value.

2. EVA – Value drivers

- Although we imposed strong assumptions about the evolution and determinants of FCFF, this simple model helps to understand the effects of the two value drivers.

2. EVA – Value drivers

- In this simple setup, to increase its value, a company must do one or more of the followings:
 1. **Increase initial NOPLAT**
 2. **Increase ROIC**
 3. **Increase Net investment rate as long as ROIC > WACC**
 4. **Reduce WACC**

2. EVA definition

- The motivation of EVA comes from the following idea:
- The value created by a company during any time period (its economic profit) must take into account not only the expenses recorded in its accounting records but also the opportunity cost of the capital employed in the business.
- This idea dates back at least to the economist Alfred Marshall, to 1890.

2. EVA definition

- This idea in other words is the following:
- The fact that net income is positive is not necessarily a good for the owners of the firm.
- Net profit must be high enough to compensate the opportunity cost of capital employed in the business.

2. EVA definition

- EVA measures the value created in a company in a single period of time and is defined as follows:

$$\text{EVA} = \text{Operating invested capital} \times (\text{ROIC} - \text{WACC})$$

Alternatively, we can also write:

$$\begin{aligned} \text{EVA} &= \text{NOPLAT} - \\ &\quad - (\text{Operating invested capital} \times \text{WACC}) \\ &= \text{NOPLAT} - \text{Capital charge} \end{aligned}$$

2. EVA definition

- An advantage of the EVA model over the DCF model is that EVA is a useful measure for understanding a company's performance in any single year, while free cash flow (FCF) is not:

2. EVA definition

- For example, you would not track a company's progress by comparing actual and projected FCF, because FCF is determined by highly discretionary investments in fixed assets and working capital.
- Management could easily delay investments simply to improve FCF in a given year at the expense of long term value creation.

2. Valuation with EVA

- The EVA model says that the value of a company equals the amount of capital invested, plus a premium or discount equal to present value of its predicted EVA:
- **Firm value =**
Operating invested capital +
Present value of predicted EVA

2. Valuation with EVA

- We follow the previous simple example to show that EVA valuation gives identical result to the DCF valuation presented before.
- See the file "EVA.xls" for the exercise.
- The **initial NOPLAT** = 100 and we impose the same assumptions as before:
- **Assumptions:** ROIC, WACC and Net investment rate are constant. Firm lives for infinite periods.

2. Valuation with EVA

- Example: DCF valuation: **Value = PV(FCFF)**

ROIC	20%				
Net investment rate	25%				
WACC	12%				
<i>year</i>	<i>1</i>	<i>2</i>	<i>3</i>	<i>4</i>	<i>5</i>
NOPLAT	100.0	105.0	110.3	115.8	121.6
Net investment	25.0	26.3	27.6	28.9	30.4
FCFF	75.0	78.8	82.7	86.8	91.2
CF growth rate, <i>g</i>		5.0%	5.0%	5.0%	5.0%
Growing perpetuity					
VALUE	1071.4				

2. Valuation with EVA

- Example: EVA valuation: **Value =**
PV(EVA) + Operating invested capital
 - Remark: In the following table, operating invested capital is computed using the definition of ROIC:
- $$\text{ROIC} = \frac{\text{NOPLAT}}{\text{Operating invested capital}}$$
- Thus, Operating invested capital = NOPLAT / ROIC.

2. Valuation with EVA

ROIC	20%				
Net investment rate	25%				
WACC	12%				
<i>year</i>	<i>1</i>	<i>2</i>	<i>3</i>	<i>4</i>	<i>5</i>
NOPLAT	100.0	105.0	110.3	115.8	121.6
Net investment	25.0	26.3	27.6	28.9	30.4
Operating invested capital	500.0	525.0	551.3	578.8	607.8
EVA	40.0	42.0	44.1	46.3	48.6
EVA growth rate, <i>g</i>		5.0%	5.0%	5.0%	5.0%
Growing perpetuity					
PV(EVA)	571.4				
Operating invested capital	500.0				
VALUE	1071.4				

2. Valuation with EVA

- To compute the present value of EVA, we use the PV formula for growing perpetuity:
Value = EVA₁ / (WACC – g)
 where EVA₁ is EVA in the first year and *g* is the EVA growth rate.
- In the "EVA.xls" file, the EVA valuation for alternative values of ROIC, WACC, and Net investment rate are presented. We can see that always the same result is obtained as in the DCF valuation.

3. Relative valuation - multiples

3. Relative valuation - multiples

- In relative valuation, the value of an asset is derived from the pricing of comparable assets, standardized using a common variable such as earnings, CFs, book value, or revenues.
- Example: Use the industry-average price/earnings ratio to value a firm, the assumption being that the other firms in the industry are comparable to the firm being valued and that the market prices these firms correctly.

3. Relative valuation - multiples

- The following ratios are being used in relative valuation:
 1. **P/E = price/earnings ratio**
 2. **Price/book value ratio**
 3. **Price/sales ratio**
 4. **Price/CF ratio**
 5. **Price/dividends ratio**
 6. **Market value/replacement value (Tobin's Q)**

3. Relative valuation - multiples

- There are two ways of using multiples:
 1. **Using fundamentals**
 2. **Using comparables**

3. Relative valuation – multiples 1. Using fundamentals

- In this approach, we relate the multiple to fundamental variables like CFs, payout ratios and risk.
- The advantage of this approach is that it shows the relationship between multiples and firm characteristics and allows us to explore how multiples change as these characteristics change:
- $(P/E)_t = \alpha + \beta(\text{Fundamental variables})_t + \varepsilon_t$

3. Relative valuation – multiples 2. Using comparables

- The second approach estimates multiples for a firm by looking at a comparable firm.
- The key issue in this approach is the definition of the comparable firm.
- The analyst should control for all variables that can influence the multiple.
- Controlling for these variables can be naive (**industry averages**) or more sophisticated (**multivariate regressions**):

3. Relative valuation – multiples
2. Using comparables

(a) Industry average approach:

$$\text{Multiple}_e = \frac{1}{k} \sum_{i=1}^k \text{Multiple}_i$$

where $i = 1, \dots, k$ are comparable firms in the sample (for example competitors).

3. Relative valuation – multiples
2. Using comparables

(b) Regression approach:

- Select n variables $\{X_{1i}, \dots, X_{ni}\}$ for firm i that influence the value of the multiple. Then, estimate the next regression model:

$$\text{Multiple}_i = \alpha + \beta_1 X_{1i} + \dots + \beta_n X_{ni} + \varepsilon_i$$

where $i = 1, \dots, k$ are comparable firms in the sample (for example competitors).

3. Relative valuation – multiples
2. Using comparables

(b) Regression approach:

- After estimating the parameters of the previous regression model, we compute the multiple of firm e using the next formula::

$$\text{Multiple}_e = \alpha + \beta_1 X_{1e} + \dots + \beta_n X_{ne}$$

where the parameters included were estimated by the regression model of the previous slide.

3. Relative valuation - multiples

- After estimating the multiple of firm e by approaches (a) or (b), one can compute the price of stock e for some multipliers in the following way:

$$(1) P_e = (P/E)_e \times E_e$$

$$(2) P_e = (P/BV)_e \times BV_e$$

$$(3) P_e = (P/Sales)_e \times Sales_e$$

3. Relative valuation – multiples
Advantages

- It is relatively easy to use.
- Multiples can be used to obtain estimates of value quickly, and are particularly useful when there are a large number of comparable firms being traded on the financial market and the market is, on average, pricing these firms correctly.

3. Relative valuation – multiples
Disadvantages

- Multiples are also easy to misuse and manipulate, especially when comparable firms are used.
- Given that no two firms are exactly similar in terms of risk and growth, the definition of comparable firms is a subjective one.
- A biased analyst can choose a group of comparable firms to confirm his/her biases about a firm's value.

3. Relative valuation – multiples Disadvantages

- While this potential for bias exists with DCF valuation as well, the analyst in DCF method is forced to be more explicit about the assumptions which determine the final value.
- With multiples, these assumptions are often left unstated.

3. Relative valuation – multiples Disadvantages

- The other problem with multiples is that it builds in errors (overvaluation or undervaluation) that the market might be making in valuing these firms.

3. Relative valuation - multiples

- **Example:** What is the value of my book shop with monthly sales of 3000 EUR if during the last month another book shop with monthly sales of 2000 EUR has been sold for 200,000 EUR?
- **Answer:** We use the Price/Sales multiple of the other shop to value our firm:
- Price/Sales = 200,000/2,000
- Value = 200,000/2,000 X 3,000 =
= 300.000 EUR

4. Contingent claim valuation

4. Contingent claim valuation Introduction

- Much work has been done in the past in developing models that value options, and these option pricing models can be used to value any asset that have option-like features. (i.e., assets that pay off under certain contingencies).
- As equity can be viewed as an option, derivative pricing models like Cox-Ross-Rubinstein or Black-Scholes can be employed to price companies.

4. Contingent claim valuation Introduction

- A **limitation** of the option pricing approach of firm valuation comes from the assumptions of the option pricing model.
- For example, volatility of the underlying asset is assumed to be constant, or other unrealistic assumptions build in errors in the pricing model.
- Thus, the final values obtained from these applications of option pricing models have much more estimation errors than more standard applications.

4. Contingent claim valuation

- On the remaining slides of this section, application of option pricing theory to **equity valuation** is presented.

4. Contingent claim valuation The general framework

- The equity in a firm is a residual claim, that is, equity holders lay claim to all CFs left over after other financial claimholders (debt, preferred stock, etc.) have been satisfied.
- The principle of **limited liability**, however, protects equity investors in publicly traded firms if the value of the firm is less than the value of the outstanding debt, and they cannot lose more than their investment in the firm.

4. Contingent claim valuation The general framework

- The **payoff to equity** investors, on liquidation, can be written as:

$$\text{Payoff} = \begin{cases} V - D & \text{if } V > D \\ 0 & \text{if } V \leq D \end{cases}$$

where V = value of the firm, D = face value of the outstanding debt and other external claims.

- Notice that a call option with spot underlying price V and strike D has the same payoff.

4. Contingent claim valuation The general framework

- Equity thus can be viewed as a **call option** on the firm value, where exercising the option requires that the firm be liquidated and the face value of the debt (which corresponds to the exercise price) be paid off.

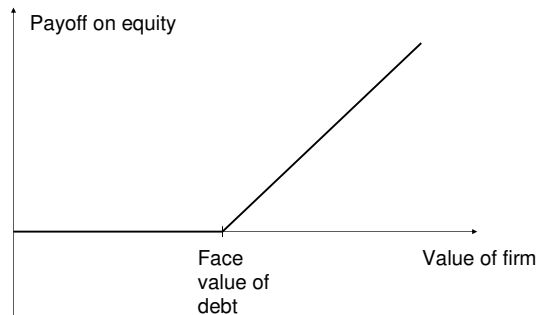
4. Contingent claim valuation The general framework

- If the debt is a single issue of zero-coupon bonds with a fixed lifetime and the firm can be liquidated by equity investors at any time prior, the life of the equity as a call option corresponds to the life of the bonds.

4. Contingent claim valuation The general framework

- The next figure presents the payoff of equity as a function of the value of the firm at liquidation:

4. Contingent claim valuation The general framework



4. Contingent claim valuation Example

- Assume a firm whose assets are currently valued at 100 million USD and that the standard deviation of this asset value is 40%.
- Further, assume that the face value of debt is 80 million USD. (It is a zero-coupon debt with ten years left to maturity.)

4. Contingent claim valuation Example

- (a) If the ten-year Treasury bond rate is 10%, how much is the equity worth?
- (b) What should the interest rate on debt be?

The use of the Black-Scholes option pricing approach provides a solution:

4. Contingent claim valuation Example

- S_0 = Value of the underlying asset = Value of firm = 100 million USD
- X = Exercise price = Face value of outstanding debt = 80 million USD
- T = Life of option = Life of zero-coupon bond = 10 years
- σ = Volatility of underlying asset = Volatility of firm value = 40%
- r = Risk-free rate = Treasury bond rate corresponding to option life = 10%

4. Contingent claim valuation Example

- (a) Based upon these inputs, the Black-Scholes model provides the following value for the call:

$$d_1 = 1.5994 \quad N(d_1) = 0.9451$$

$$d_2 = 0.3345 \quad N(d_2) = 0.6310$$
- Value of call = $S_0 N(d_1) - X \exp(-rT)N(d_2)$
 $= (100 \times 0.9451) - 80 \times \exp(-0.1 \times 10) \times 0.6310$
 $= 75.94 \text{ million} = \text{value of equity}$

4. Contingent claim valuation Example

- (b) Value of outstanding debt = $100 - 75.94 = 24.06$ million
- Interest on debt = $(80/24.06)^{1/10} - 1 = 12.77\%$

4. Contingent claim valuation Some valuable insights

- There are some valuable insights that can be obtained by viewing equity as a call option:
 - (1) **Valuing equity in a troubled firm**
 - (2) **The conflict between bondholders and stock holders**

4. Contingent claim valuation Valuing equity in a troubled firm

- The first implication is that equity will have value, even if the value of the firm falls well below the face value of the outstanding debt.
- Such a firm will be viewed as troubled by investors, accountants, and analysts, but that does not mean that its equity is worthless.

4. Contingent claim valuation Valuing equity in a troubled firm

- In fact, just as deep out-of-the money traded options command value because of the possibility that the value of the assets may increase above the face value of the bonds before they come due.

4. Contingent claim valuation The conflict between bondholders and stock holders

- Stockholders and bondholders have different objective functions, and this can lead to agency problems, where stockholders can expropriate wealth from bondholders.
- The conflict can manifest itself in a number of ways – for instance, stockholders have an incentive to take riskier projects than bondholders do.

4. Contingent claim valuation The conflict between bondholders and stock holders

- This conflict between bondholders and stockholders can be illustrated dramatically using the option pricing model.
- Since equity is a call option on the value of the firm, an increase in the variance in the firm value, other things remaining equal, will lead to an increase in the value of equity.

4. Contingent claim valuation The conflict between bondholders and stock holders

- It is therefore conceivable that stockholders can take risky projects with negative present values, which, while making them better off, may make the bondholders and the firm less valuable.

4. Contingent claim valuation
Main assumptions

Previously, we made some simplifying assumptions:

1. There are only two claimholders in the firm – debt and equity.
2. There is only one issue of debt outstanding, and it can be retired at face value.
3. The debt has a zero-coupon and no special features (convertibility, etc.)
4. The value of the firm and its variance can be estimated.

4. Contingent claim valuation
Practical use of this model

- These assumptions we made to have a more simple problem of valuation.
- However, most firms do not fall into the neat framework developed above of having only one zero-coupon bond outstanding, there are compromises that have to be made to use this model in valuation.
- In the followings, we present in some points how the option pricing model can be applied in practice.

4. Contingent claim valuation
Practical use of this model

1. Value of the firm:

- The value of the firm can be obtained in two ways:
- (a) The market values of outstanding debt and equity are summed, assuming that all debt and equity are traded, to obtain firm value.
- (b) The market values of assets are estimated, either by discounting expected CFs at the WACC or by using prices from a market that exists for these assets.

4. Contingent claim valuation
Practical use of this model

2. Variance of firm value:

- The variance in firm value can be obtained directly if both stocks and bonds in the firm trade in the marketplace.

4. Contingent claim valuation
Practical use of this model

- The variance in firm value can be written as:

$$\sigma_{\text{firm}}^2 = w_e^2 \sigma_e^2 + w_d^2 \sigma_d^2 + 2w_e w_d \rho_{ed} \sigma_e \sigma_d$$

where σ_e and σ_d are the volatility of equity and debt price returns, w_e and w_d are the market-value weights of equity and debt, respectively. Finally, ρ_{ed} denotes the correlation coefficient between equity and debt returns.

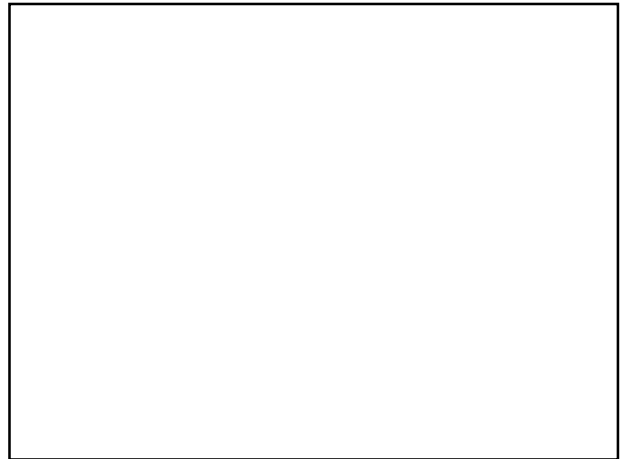
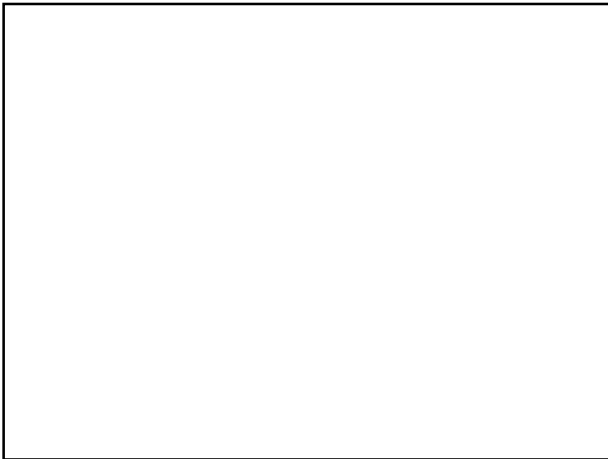
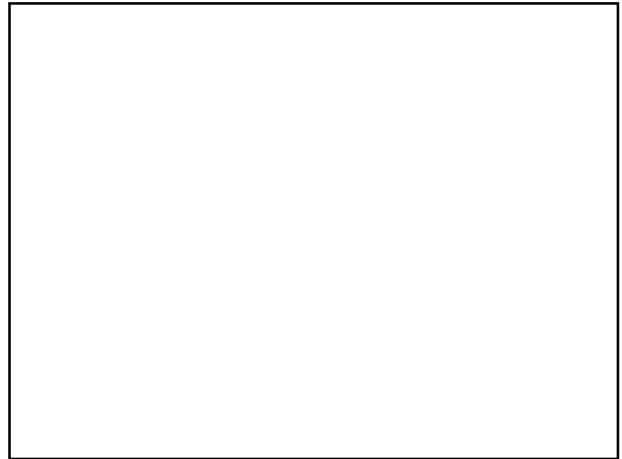
4. Contingent claim valuation
Practical use of this model

3. Maturity of debt:

- In contrast with the assumption that a firm has one zero-coupon bond outstanding, most firms have more than one debt issue on their books, and much of the debt comes with coupons.
- Since the option pricing model allows for only one input for the time to expiration, these multiple bonds issues and coupon payments have to be compressed into one measure.

4. Contingent claim valuation
Practical use of this model

- One solution is that take into account both the coupon payments and the maturity of the bonds is to estimate the **duration** of each debt issue and calculate a **face-value-weighted average of the durations** of the different issues.
- This **value-weighted duration** is then used as a **measure of the time to expiration of the option**.



COCA-COLA DCF VALUATION

COCA-COLA DCF CASE

STRUCTURE:

- (1) Introduction
- (2) Company report of Coca-Cola
- (3) Historical summary and ratio analysis
- (4) Comparison with other firms
- (5) Forecasting free cash flow
- (6) Cost of capital estimation
- (7) DCF valuation of equity and assets

(1) Introduction

Introduction

- In this class, the DCF valuation technique is applied for a private firm.
- We value a U.S. company: Coca-Cola.

Why Coca-Cola is selected?

- Coca-Cola has a **homogenous product structure** (soft drinks):
- Most of the sales is obtained by its main product: the **Coca-Cola drink**.
- This is a large company and **main accounting documents** necessary for DCF valuation are **available**.

Why Coca-Cola is selected?

- The ratio of operational costs/sales is **stable** over time (see later).
- The firm production is **relatively simple** from a **technological point of view**.
- Several accounts of the balance sheet maintain a **stable proportion** to total assets over time (see later).
- These last three points help in more precise forecasting of future CFs.

Why Coca-Cola is selected?

- Coca-Cola is **publicly traded on the stock exchange**. Therefore, the cost of capital can be computed using the capital asset pricing model (CAPM) or other valuation models of finance.
- Coca-Cola is an **old, stable company**. Therefore, we can reach the steady state continuous value period by forecasting only few explicit forecast periods.

A remark about financial documents

- In firm valuation, we always need the main **accounting documents** of the company.
- Suppose that we do not have a company report data base purchased.
- How can we get accounting document data on firms though **internet**?

A remark about financial documents

- For U.S. firms, one possibility is to go to the www.msn.com website.
- Select the link **"Money"**.
- Write the name of the firm or its quote symbol into the **"Get quote"** box.
- For the Coca-Cola, the quote symbol is **"KO"**.

A remark about financial documents

- In the **"Fundamentals"** section, we can choose the links:
 - (1) company report or
 - (2) financial results.
- In the (2) financial results part, there are **Income statement, Balance sheet, Cash flow statement** and **key financial ratios** available for the last five years.

A remark about financial documents

- Other web-based sources of U.S. Financial documents are
 - (1) www.google.com/finance
- Write the name of the firm or its quote symbol in the search box.
- (2) www.yahoo.com
- Choose the link "Finance"
- Write the name of the firm or its quote symbol into the **"Get quote"** box.

(2) Company report of Coca-Cola

Company report of Coca-Cola

- The Coca-Cola Company is the owner and marketer of **nonalcoholic beverage brands**.
- It also manufactures, distributes and markets **concentrates** and **syrops** used **to produce nonalcoholic beverages**.

Company report of Coca-Cola

- The Company owns or licenses and markets more than 500 nonalcoholic beverage brands, primarily **sparkling beverages** but also a **variety of still beverages**, such as waters, enhanced waters, juices and juice drinks, ready-to-drink teas and coffees, and energy and sports drinks.

Company report of Coca-Cola

- It also manufactures, or authorizes bottling partners to manufacture, **fountain syrups**, which it sells to fountain retailers, such as restaurants and convenience stores, which use the fountain syrups **to produce finished beverages for immediate consumption**.

Company report of Coca-Cola

- See more information on the company website:
<http://www.thecoca-colacompany.com/>

(3a) Historical summary

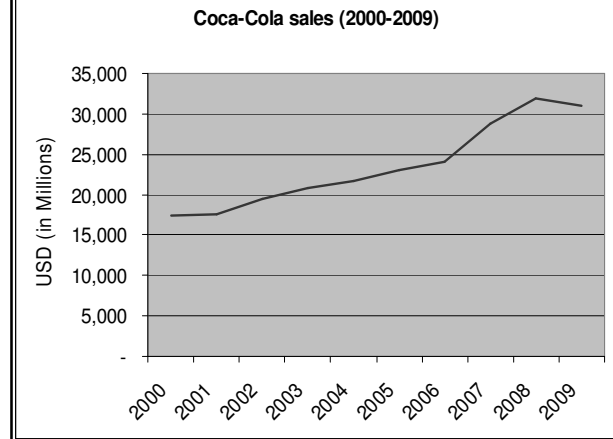
Historical summary and ratios

- On the following slides, some key financial figures of Coca-Cola are presented over the 2000-2009 period.
- First, we present some figures from the **Income statement**.
- Then, we look at some items of the **Balance sheet** of Coca-Cola.

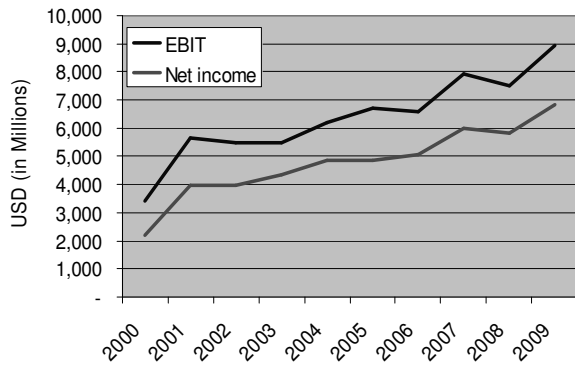
Income Statement: 10 Year Summary

(in Millions USD)

year	Sales	EBIT	Net Income	Tax Rate (%)
2000	17,354	3,399	2,177	35.95
2001	17,545	5,670	3,979	29.82
2002	19,564	5,499	3,976	27.70
2003	20,857	5,495	4,347	20.89
2004	21,742	6,222	4,847	22.10
2005	23,104	6,690	4,872	27.17
2006	24,088	6,578	5,080	22.77
2007	28,857	7,919	5,981	23.89
2008	31,944	7,506	5,807	21.74
2009	30,990	8,946	6,824	22.80



Profit figures of Coca-Cola for (2000-2009)



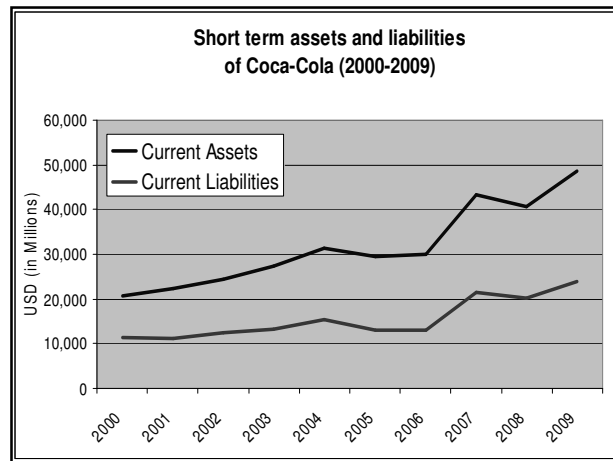
Summary of Income statement figures

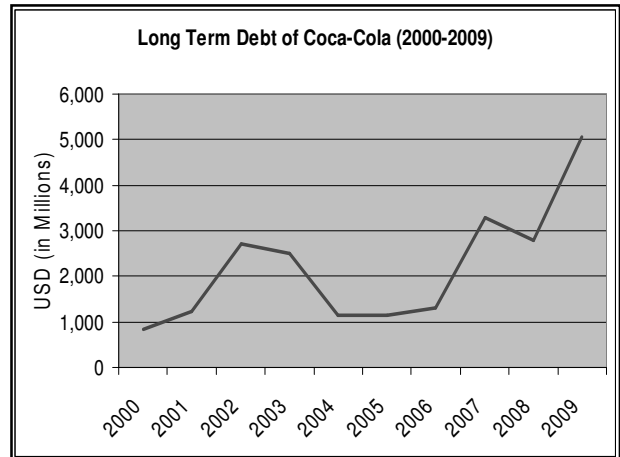
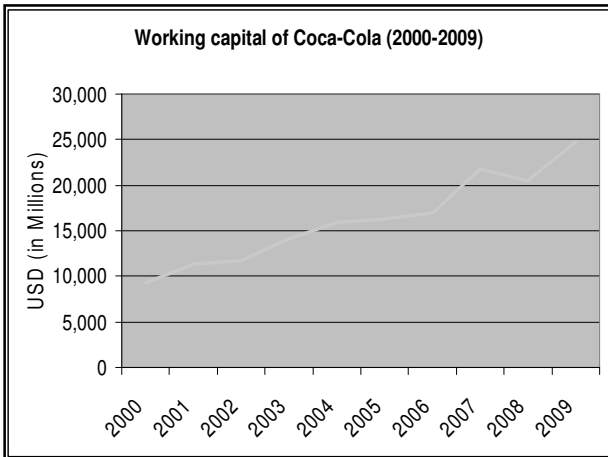
- We can see from these figures of Coca-Cola that **sales** and both **alternative profit measures** have been **increasing** during the past ten years.

Balance Sheet: 10 Year Summary

(in Millions USD)

year	Current Assets	Current Liabilities	Working capital	Long Term Debt
2000	20,834	11,518	9,316	835
2001	22,417	11,051	11,366	1,219
2002	24,406	12,606	11,800	2,701
2003	27,342	13,252	14,090	2,517
2004	31,441	15,506	15,935	1,157
2005	29,427	13,072	16,355	1,154
2006	29,963	13,043	16,920	1,314
2007	43,269	21,525	21,744	3,277
2008	40,519	20,047	20,472	2,781
2009	48,671	23,872	24,799	5,059





Summary of Balance sheet figures

- From the balance sheet figures, we can see that both current assets and liabilities have had a stable, **increasing** tendency during the last years.
- Moreover, working capital of Coca-Cola has been positive and **increasing** during the last decade.
- These are signs of a **stable growth** of the company's operations.

Summary of Balance sheet figures

- On the last figure, we can see that the **long-term financing** of Coca-Cola has been **increasing** in general during the last years.
- There was an especially significant growth in long-term liabilities during the **last four years** (2006-2009).

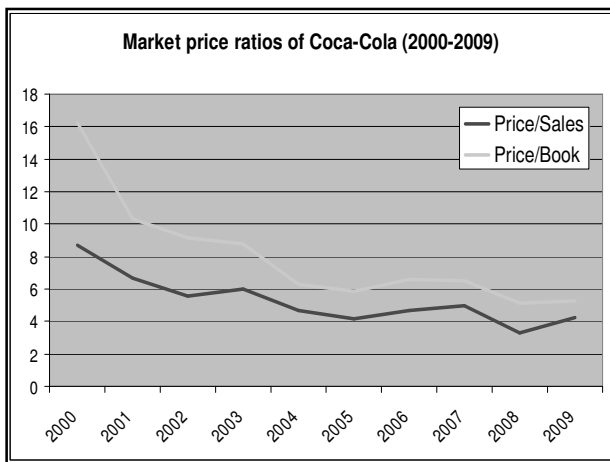
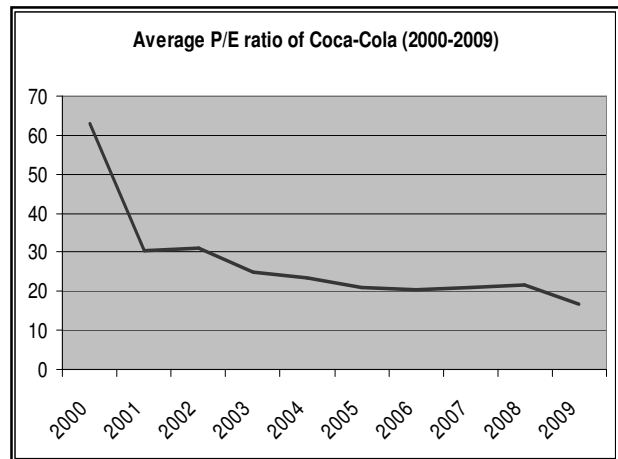
(3b) Ratio analysis

Ratio analysis

- On the following slides, the historical evolution of several financial ratios of Coca-Cola is reviewed for the 2000-2009 period.
- We will see the evolution of
 - (1) **Market price ratios**
 - (2) **Profitability ratios**
 - (3) **Leverage ratios**

(1) Market price ratios

year	Average P/E	Price/Sales	Price/Book
2000	63	8.7	16.25
2001	30.4	6.68	10.31
2002	31.1	5.56	9.18
2003	25	5.99	8.79
2004	23.3	4.65	6.3
2005	21	4.18	5.84
2006	20.3	4.71	6.61
2007	21	4.96	6.54
2008	21.6	3.31	5.11
2009	16.7	4.28	5.29

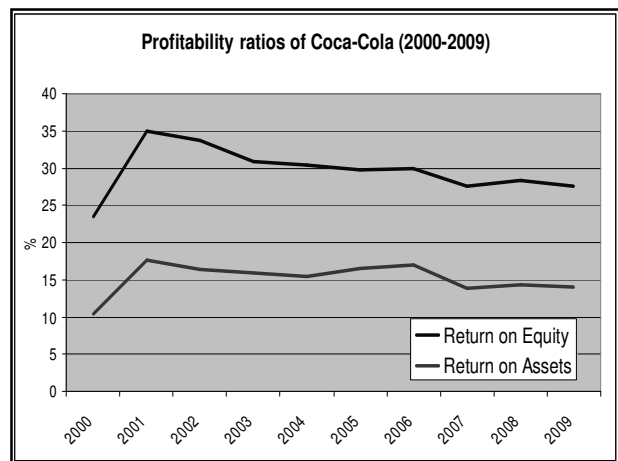


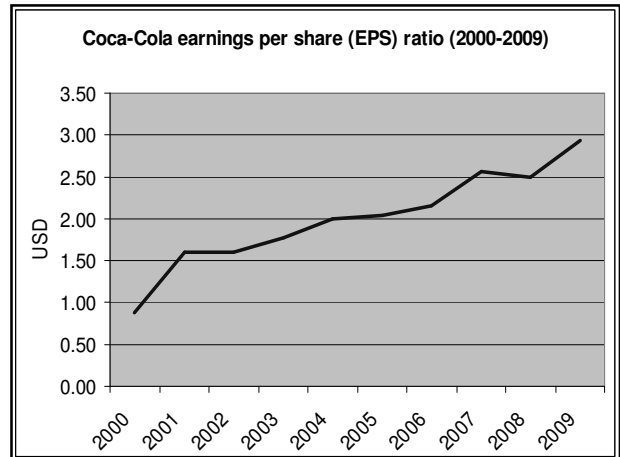
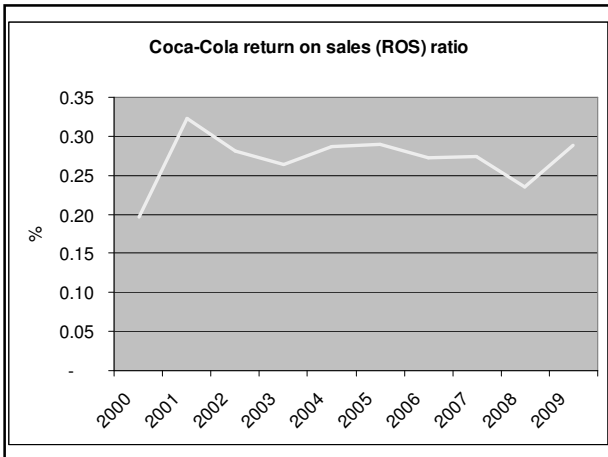
Summary of Market price ratios

- We can see that the P/E, P/Sales and P/Book value ratios of Coca-Cola have been decreasing during 2000-2009.
- **Interpretation:** Most analysts believe that **low P/E** stocks are more attractive than high P/E stocks because they allow the investor to **pay less USD of current earnings**.
- The same positive conclusion can be obtained looking at the P/Sales and P/Book value ratios.

(2) Profitability ratios

year	ROE	ROA	ROS	EPS
2000	23.40	10.40	0.20	0.88
2001	35.00	17.70	0.32	1.60
2002	33.70	16.30	0.28	1.60
2003	30.90	15.90	0.26	1.77
2004	30.40	15.40	0.29	2.00
2005	29.80	16.60	0.29	2.04
2006	30.00	17.00	0.27	2.16
2007	27.50	13.80	0.27	2.57
2008	28.40	14.30	0.23	2.49
2009	27.50	14.00	0.29	2.93





Summary of profitability ratios

- We can see that ROE and ROA have been decreasing and ROS has been relatively stable during 2000-2009.
- In the same time, EPS has been increasing significantly.
- The last point seems to be contradictory but remember the way ROE and EPS are computed:

$$\text{ROE} = \text{Net income} / \text{Equity}$$

$$\text{EPS} = \text{Net income} / \# \text{ common shares}$$

Summary of profitability ratios

- Previously, we saw that Coca-Cola net income increased very significantly from 2,177 Millions USD to 6,824 Millions USD.
- The evolution of ROE can be explained by the fact that the denominator of ROE (i.e., equity) was increasing faster than Net income.
- The significant evolution of EPS is also motivated by the fact that the denominator of EPS (i.e., number of common shares outstanding) was significantly decreasing.

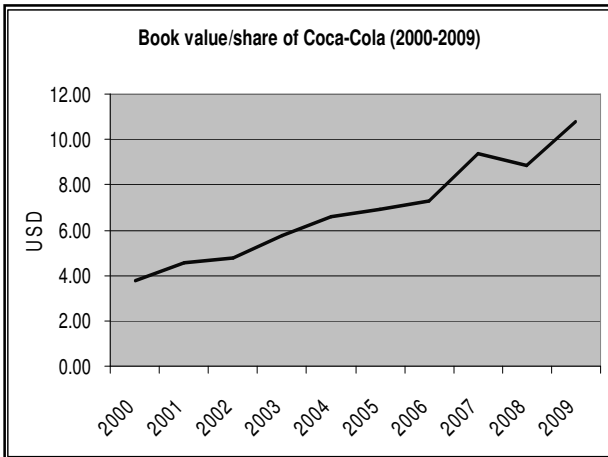
Summary of profitability ratios

- The decreasing tendency of number of common shares can be explained by the fact that Coca-Cola repurchased own shares on the market during the last years.
- These shares are represented in the Equity section of Balance with negative values on the following account:
- Treasury stock – Common (Millions USD):

31/12/2005	31/12/2006	31/12/2007	31/12/2008	31/12/2009
(19,644)	(22,118)	(23,375)	(24,213)	(25,398)

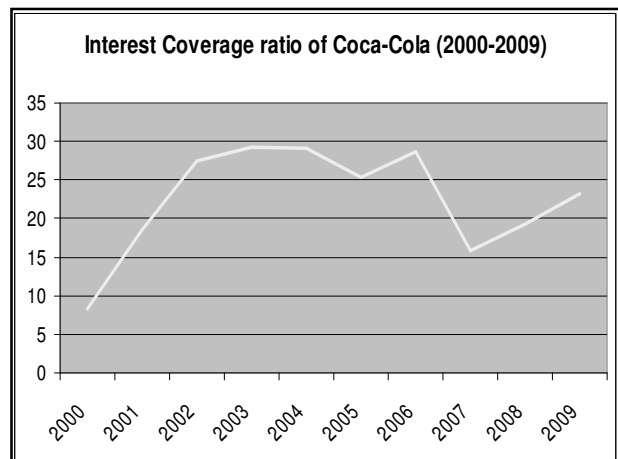
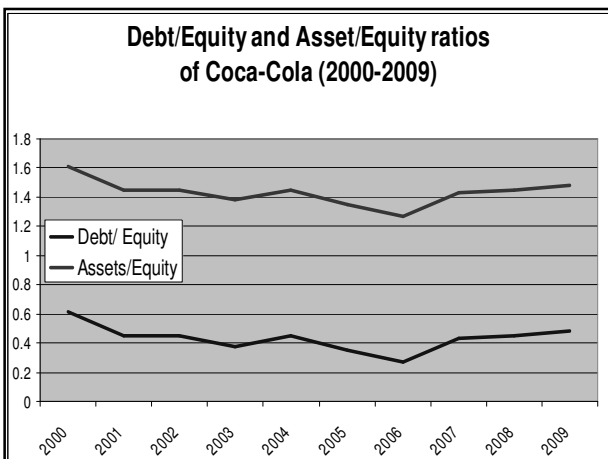
Summary of profitability ratios

- This own share repurchase resulted in a **concentration of common shares** among outstanding shareholders. →
- It increased the net income per shares outstanding (i.e., **EPS increased**).
- Another consequence of repurchasing own shares has been the **increasing** book value per common share presented on the next figure:



(3) Leverage ratios

year	Debt/ Equity	Leverage = Assets/Equity	Interest Coverage
2000	0.61	1.61	8.3
2001	0.45	1.45	18.5
2002	0.45	1.45	27.4
2003	0.38	1.38	29.3
2004	0.45	1.45	29.1
2005	0.35	1.35	25.4
2006	0.27	1.27	28.7
2007	0.43	1.43	15.9
2008	0.45	1.45	19.3
2009	0.48	1.48	23.2



Summary of leverage ratios

- The leverage ratio defined as

$$\text{Leverage} = \text{assets/equity} = 1 + \text{debt/equity}$$
- We can see that Coca-Cola's leverage has been **decreasing** during 2000-2005.
- However, during 2006-2009 the firm's leverage **increased**.

Summary of leverage ratios

- From year 2000, the interest coverage of Coca-Cola has **increased** significantly until 2002.
- During the 2003-2006 period interest coverage has been relatively **stable**.
- After 2006, interest coverage of Coca-Cola has **decreased** significantly.

(4) Comparison with other firms

Comparison of financial ratios

- On the following slides, we compare several ratios of Coca-Cola to the
 - (1) **Industry average** and
 - (2) **S&P500 firms' average**.
- This way we can compare various aspects of Coca-Cola financial performance to that of its competitors and other firms in the U.S. economy represented by the S&P500 index.

Comparison of financial ratios

- We review several ratios in the following structure:
 - (1) **Growth rates of some accounts of the Income statement**
 - (2) **Profitability ratios**
 - (3) **Profit margins**
 - (4) **Market price ratios**
 - (5) **Leverage and liquidity ratios**
 - (6) **Efficiency ratios**

Comparison of financial ratios

- In the following tables, we can observe several ratios for each group.
- **Note:** When the exact time of the ratio is not specified clearly then it will mean the last year (2009) figures.

Growth rates (Income statement)

Growth Rates %	Company	Industry	S&P 500
Sales	5.4	6	10.7
Net Income	17.5	18.5	14.3
Sales (5-Year Annual Avg.)	7.35	8.84	7.75
Net Income (5-Year Annual Avg.)	7.08	8.99	8.24
Dividends (5-Year Annual Avg.)	10.4	8.93	4.97

Profitability ratios

Investment Returns %	Company	Industry	S&P 500
ROE	30.2	38.8	20.7
ROA	15.5	12.7	6.9
ROIC	22.4	18.5	9.4
ROE (5-Year Average)	29.8	22.5	16.2
ROA (5-Year Average)	15.7	12.3	7.6
ROIC (5-Year Average)	23	17.8	10.3

Profitability ratios

Definitions of profitability ratios:

- **ROE = Net income / Equity**
- **ROA = EBIT / Total assets**
- **ROIC = return on operating invested capital = NOPLAT / Operating invested capital**

Profit margins

Profit Margins %	Company	Industry	S&P 500
Gross Margin	64.2	58.5	38.3
Pre-Tax Margin	28.9	22.9	14.2
Net Profit Margin	22.3	17.7	10.5
5Yr Gross Margin (5-Year Avg.)	64.6	58.8	37.9
5Yr Pre-Tax Margin (5-Year Avg.)	27.1	21.1	15.9
5Yr Net Profit Margin (5-Year Avg.)	20.7	16	11.3

Profit margins

Definitions of profit margins:

- **Gross margin = EBIT / Sales**
- **Pre-tax margin = Pre-tax profit / Sales**
- **Net profit margin = Net income / Sales**

Market price ratios

Price Ratios	Company	Industry	S&P 500
Current P/E Ratio	18.8	18.8	21.7
Price/Sales Ratio	4.09	3.3	2.28
Price/Book Value	5.1	5.52	5.82
Price/Cash Flow Ratio	15.6	14.1	16.4

Market price ratios

Definitions of market price ratios:

- **P/E = price / earnings per share**
- **P/Sales = price / sales per share**
- **P/Book value = price / book value per share**
- **P/Cash flow = price / FCFE**

Leverage and liquidity ratios

Financial Condition	Company	Industry	S&P 500
Debt/Equity Ratio	0.48	1.27	1.39
Current Ratio	1.3	1.4	1.4
Quick Ratio	1.1	1.1	1.2
Interest Coverage	NA	2.4	27.2
Leverage Ratio	2	3.4	4.5
Book Value/Share	10.77	11.08	22.06

Leverage and liquidity ratios

Definitions of leverage and liquidity ratios:

- **Current ratio = current assets / current liabilities**
- **Quick ratio = (cash + marketable securities + receivables) / current liabilities**
- **Interest coverage = EBIT / interest expense**
- **Leverage = assets / equity**

Efficiency ratios

Management Efficiency	Company	Industry	S&P 500
Net income/Employee	74,418	62,521	75,397
Sales/Employee	333,944	334,062	814,514
Receivable Turnover	9.1	10.2	14.4
Inventory Turnover	4.9	6.7	9.2
Asset Turnover	0.7	0.8	0.8

Efficiency ratios

Definitions of efficiency ratios:

- **Receivable turnover = Cost of goods sold / receivables**
- **Inventory turnover = Cost of goods sold / inventories**
- **Asset turnover = Sales / total assets**

(5) Forecasting FCF

Forecasting FCF

- Using accounting information downloaded from www.msn.com, the Balance sheets and Income statements were constructed for the last five years: 2005-2009.

Forecasting FCF

- The following three slides show the structure of the:

- (1) **Balance sheet: Assets**
- (2) **Balance sheet: Liabilities and equity**
- (3) **Income statement**

Assets
Total Current Assets
Cash and Short Term Investments
Total Receivables, Net
Total Inventory
Prepaid Expenses
Other Current Assets, Total
Total fixed assets
Property/Plant/Equipment, Total - Net
Goodwill, Net
Intangibles, Net
Long Term Investments
Note Receivable - Long Term
Other Long Term Assets, Total
Other Assets, Total

Liabilities and Shareholders' Equity
Total liabilities
Total Current Liabilities
Accounts Payable
Payable/Accrued
Accrued Expenses
Notes Payable/Short Term Debt
Current Port. of LT Debt/Capital Leases
Other Current Liabilities, Total
Total long-term liabilities
Total Long Term Debt
Deferred Income Tax
Minority Interest
Other Liabilities, Total
Total Equity
Redeemable Preferred Stock
Preferred Stock - Non Redeemable, Net
Common Stock
Additional Paid-In Capital
Retained Earnings (Accumulated Deficit)
Treasury Stock - Common
Other Equity, Total

Total Revenue
Cost of Revenue, Total
Gross Profit
Selling/General/Administrative Expenses, Total
Research & Development
Interest Expense (Income), Net Operating
Other Operating Expenses, Total
EBITDA
Depreciation/Amortization
EBIT
Interest Income (Expense), Net Non-Operating
Financial result
Unusual Expense (Income)
Gain (Loss) on Sale of Assets
Other, Net
Extraordinary result
Income Before Tax
Income Tax - Total
Minority Interest
Equity In Affiliates
U.S. GAAP Adjustment
Net Income

Forecasting FCF

- We shall forecast the cash flow statement as follows:
- First, we forecast the Balance sheet and the Income statement for six years.
- The first five years define the explicit forecast period.
- The sixth year is the starting year of the continuing value period.

Forecasting Balance sheet

- To forecast the Balance sheet, we simply computed the percentage of each account to total assets during the past 5 years.
- Then, we identified the relatively stable percentages and computed the average percentage of the account over the stable period.

Forecasting Balance sheet

- Then, we assumed that the value of total assets will grow according to the GDP growth rate of the U.S.
- We assumed 3 percent for the U.S. growth rate and forecasted total assets for the next six years.
- Finally, we used the percentage averages of single accounts to compute the forecasted value of each account in the Balance sheet.

Forecasting Income statement

- In order to forecast the Income statement, we computed the following values from the Income statement:

RATIOS:	2005	2006	2007	2008	2009	mean
Total revenue(t)/total revenue(t-1)		0.04	0.20	0.11	(0.03)	0.08
Cost of revenue / total revenue	0.35	0.34	0.36	0.36	0.36	0.35
Other costs / Total revenue	0.38	0.39	0.39	0.37	0.37	0.38
Computed tax rate	0.27	0.23	0.24	0.23	0.24	0.24

Forecasting Income statement

- In the previous table, the last column contains the average of the past five years.
- We used this table to forecast the income statement for the next six years.

Forecasting FCF

- Second, for each year of the forecast period, we compute the cash flow statement.
- We use the “**type two**” CF statement learned in the Financial Statements class.
- The structure of the CF statement is the following:

	EBIT
-	Tax due to operating result
	NOPLAT
+	Depreciation/Amortization
-	Δ Current assets (without cash)
+	Δ Current liabilities
	Cash provided by operations
-	Δ Fixed assets (without financial)
-	Depreciation/Amortization
	Cash from investments
+	Extraordinary result
-	Tax due to extraordinary result
	Extraordinary cash flow
	FCFF
-	Δ Long-term financial assets
+	Δ Long-term financial liabilities
+	Financial result
-	Tax due to financial result
	Cash from financing activities
	FCFE
+	Δ Equity
-	Net Income
	Cash and Short Term Investments

Forecasting FCF

- Once we have obtained the forecasts of FCFE and FCFF we can discount these estimates to compute the values of equity and total assets, respectively.

(6) Cost of capital

Cost of capital of equity

- First, we compute the **cost of equity** of Coca-Cola using the **CAPM model**:

$$r_E = r_f + \beta_E (r_M - r_f)$$

- The analysts' report on www.msn.com tells us that **beta = 0.6**
- We also evaluate the yield to maturity of U.S. T-bills and obtain an estimate of the risk-free rate: $r_f = 5\%$
- We take the annual S&P500 return as market return: $r_m = 25\%$

Cost of capital of equity

- Then, using the CAPM we get that the cost of capital of equity is $r_E = 17\%$

Cost of capital of firm

- Second, we compute after-tax WACC using the following formula:

$$\text{WACC after taxes} = r_E \frac{E}{E+D} + r_D (1 - T_c) \frac{D}{E+D}$$

- We assume that the value of equity and debt in the Balance sheet represent **market values**.

Cost of capital of firm

- We use the tax rate in the WACC formula.
- To estimate the cost of capital of debt we compute the yield to maturity of publicly traded corporate bonds of Coca-Cola.
- We get that $r_D = 13\%$ is the cost of capital of debt.
- Then, substituting in the WACC formula we get the cost of capital of the firm:
WACC = 13.5%

(7) DCF valuation of equity and assets

DCF valuation

- We discount forecasted FCFF by WACC to get the value of total assets.
- We discount forecasted FCFE by r_E to get the value of equity.

DCF valuation

- For the **explicit forecast period** the next present value formulas are used:

$$PV(FCFE_t) = FCFE_t / (1+r_E)^t$$

$$PV(FCFF_t) = FCFF_t / (1+WACC)^t$$

DCF valuation

- For the **continuing value forecast period** we assume that **CF forms a growing perpetuity**.
- The growth rate, g is assumed to be 1.5%
- We use the following formula to compute the present value of the growing perpetuities:

$$PV(\text{continuing value}) = CF_6 / [(r-g)(1+r)^5]$$

DCF valuation

- As a summary, we present the present value formulas for the FCFE and FCFF cash flows:

$$PV(\text{TOTAL: FCFE}) = \sum_{t=1}^5 \frac{FCFE_t}{(1+r_E)^t} + \frac{FCFE_6}{[(r_E - g)(1+r_E)^5]}$$

$$PV(\text{TOTAL: FCFF}) = \sum_{t=1}^5 \frac{FCFF_t}{(1+WACC)^t} + \frac{FCFF_6}{[(WACC - g)(1+WACC)^5]}$$

DCF valuation: Summary

- We can compare to DCF valuation result with the balance sheet items in the following table:

SUMMARY:	DCF	BALANCE
Value of equity	48,865	24,799
Value of assets	65,166	48,671
Value of debt	16,302	23,872