# Avoiding clogs: The shape of arches and their stability against vibrations

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**Abstract.** A distinctive feature of discrete solids is their ability to form arches. These mechanically stable structures alter the isotropy of granular packings and can arrest the motion of grains when, for example, they flow through a bottleneck. Breaking arches can be achieved by means of an external vibration, which effectively eliminates clogging. Indeed, these phenomena and procedures are quite common in industrial applications. Nevertheless, there are not rigorous, well founded criteria to determine the most efficient way to break arches and restore the flow of grains. This happens in part because it is not known which are the relevant characteristics that boost the arch strength. In the experiment presented here, we have carried out a statistical analysis of the arches that block the exit orifice at the bottom of a two dimensional silo, and described their geometrical properties. We then submit the silo to an external vibration. We find that the larger the outlet size, the weaker the arches that clog it. This dependence is just the outcome of a more complicated process that involves geometrical defects in the arch. The defects are quantitatively defined in terms of contact angles and we show that this is a key factor regarding the endurance of arches.

**Keywords:** Clogging, jamming, arch formation, vibration, silos
**PACS:** 45.70.Mg

## INTRODUCTION

Granular flows are prone to clogging. Whenever particulate matter must pass through a bottleneck, there is the possibility that an arch forms and brings the grains to a halt. The exit of a silo discharging by gravity, for instance, is one situation where the formation of such arches has received considerable attention [1, 2, 3, 4, 5]. As these structures are mechanically stable, simply increasing the load does not promote the flow resumption. Instead, the nuisance can be dealt with by imparting a vibration, so that arches are broken. The method is widely implemented in many different industrial procedures, ranging from crude to sophisticated [6]. Theoretical and experimental analyses have been carried out to study the effect of vibrations on the granular flow through a bottleneck [7, 8, 9]. Nevertheless, the mechanism of arch breaking has not been examined in detail. A better understanding of the phenomenon can undeniably help to optimize the procedure.

Available reports on vibrated silos mostly focus on the effects that external perturbations produce on the mass flow rate. For instance, some measurements [10] reveal that the flow at high accelerations is reduced a little, although in this case the air flow is relevant. Later studies [11] also find very small influence of the vibration on the mass flow rate. In another experiment, different flow regimes have been observed in a two-dimensional hopper. The regimes could be described as jammed, intermittent and uninterrupted flow [12].

We have studied this phenomenon in the laboratory, with the aim of providing some hints that could lead to a better operation. Specifically, we measure the vibration intensity needed to break the arches that form at the exit orifice of a two-dimensional silo. The level of vibration is given by $\Gamma$, which is the amplitude of the sinusoidal acceleration divided by the gravity $g$. The first noticeable fact (shown in Fig.1) is that $\Gamma$ decreases with the orifice size $R$ (where $R$ is the length of the orifice measured in bead diameters). This makes sense, because bigger orifices need larger arches to block them, and intuitively it seems reasonable that larger arches can be shattered more easily. But an exploration of the issue shows that there is more than meets the eye. We will show that there is a complex link between the geometry of an arch and its endurance, in which geometrical defects of the arch play a prominent role.

Our experimental device consists of a two-dimensional transparent silo with an exit orifice at the bottom, placed on top of an electromagnetic shaker. This allows us to analyze the shape of the arch that eventually blocks the orifice, as well as to measure the vibration intensity needed to break it. The experiment is fully automated, thus allowing us to collect large amounts of data. The protocol is the following. About ten thousand metal beads of diameter $d = 1$ $mm$ are stored in a plane silo so that they are arranged in a monolayer. They begin to fall through the orifice at the bottom and are collected in another chamber. The formation of an arch that blocks the orifice and stops the outpouring
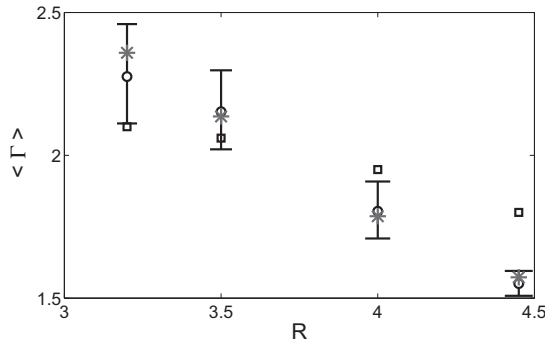
**FIGURE 1.** Experimental results of the average acceleration $\Gamma$ at which arches break, as a function of the orifice size $R$ (*circles and error bars*). The meaning of the other symbols is explained in the text.



**FIGURE 2.** The average number of beads $\eta$ in the arch that blocks an orifice of a given size. The reason for the linear fit (*dashed line*) is argued in [14].

is detected by means of a standard video camera. A photograph of the arch is then taken and stored in the computer, and a vibration ramp is applied to the shaker. The frequency of the vibration is fixed at $f = 1\ kHz$ and the amplitude is increased linearly at a rate of $0.09\ g/s$. The breaking of the arch is again detected by image analysis. At this moment, the acceleration $\Gamma$ is recorded. An electric motor then turns the assembly bottom up so that the beads fallen in the bottom chamber refill the silo. The experiment is restarted again with another half turn, when the beads from the filled silo again begin to fall through the exit orifice. A more detailed description can be found in [13].

The aim of this research is to establish a relationship between the shape of the arches and the vibration needed to break them. To this end, we will first give some results about the geometry of the arches that block the orifice. Then we will present the acceleration measurements, in which the vibration level needed to break the arch is registered. The conclusions gathered from these measurements will be used to shed new light on the data presented in Fig.1.

## ARCH GEOMETRY

The first noticeable fact concerning the geometry of the arches is that there is a direct relationship between the size of the orifice $R$ and the size of the arch that blocks it. Obviously, as $R$ is increased a larger number of beads is needed to build an arch spanning over the orifice. This is illustrated in Fig. 2, where the size of the arch, measured in number of beads $\eta$, is plotted versus $R$. Moreover, for the range of $R$ explored in this work, we had previously reported that there is a direct relationship between different geometrical properties of the arch (such as the span
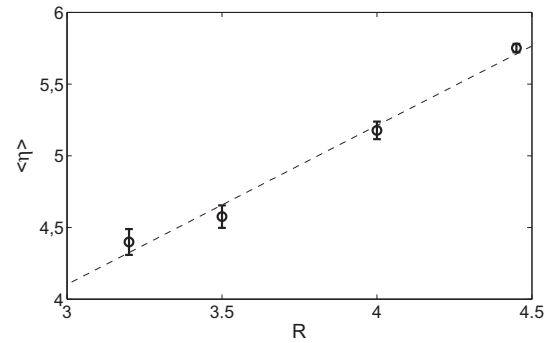
and the height) and $\eta$. Then, the study of this quantity can be taken as a figure representing the arch geometry instead of considering all the possible variables [14].

As explained above, once the arch has been photographed, a vibration ramp is switched on. The shattering of an arch is too fast to appreciate the fine details with the naked eye. An interesting surprise came when inspecting the process with a high speed camera. We recorded and analyzed two hundred arches in the moments just before they break down [15]. Most of the arches break at a *defect*. We call a defect a bead that is hanging from their neighbors due, of course, to friction: in a defect, the contact points of a bead with its neighbors are above the horizontal sphere equator. In order to quantify the importance of the defect, we defined an angle $\phi$ for each bead in the arch that is determined by the segments joining the bead center with the centers of its neighbors (therefore passing through the contact points). If this angle is larger than $180°$ then the bead is considered a defect; see Fig. 3 (a).

There are proportionately many defects: in an experimental run we have tallied that from among thirty thousand beads forming arches, about 17% are defects. And they appear evenly distributed along the arch, as shown in Fig. 3 (b). We have also obtained the angle distribution (the PDF) of all the beads in the arches for different values of $R$ [Fig. 3 (c)]. As can be seen, the angle distribution is quite similar irrespective of the orifice size, except for small values of $\phi$. This is easily understood if one thinks that beads arranged in the smallest possible disposition to block the orifice, will have comparatively smaller angles for smaller orifices size. Apart from this, the PDF seems to be robust and general.

If defects are uniformly distributed, that is, if the probability of a bead forming a defect is a fixed value, irrespective of time and space, as suggested by the results presented above, then it is justified to think of each bead
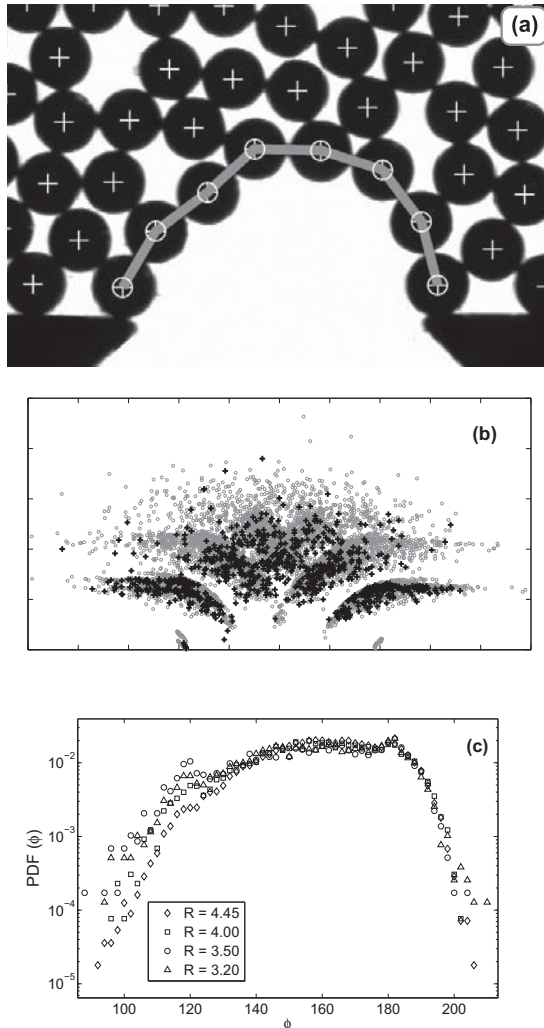
**FIGURE 3.** (a) A photograph showing an arch (*circles and gray line*) with a defect (third bead from the left). (b) A chart showing the locations of the beads in the arches (*gray circles*) and the positions of the defects *black crosses*. The map covers a region of 10x5 mm (the horizontal axis is centered on the orifice; ticks spaced 1 mm). (c) PDF of the angles corresponding to the beads in all the arches observed for four different values of $R$. Note the logarithmic scale on the vertical axis.
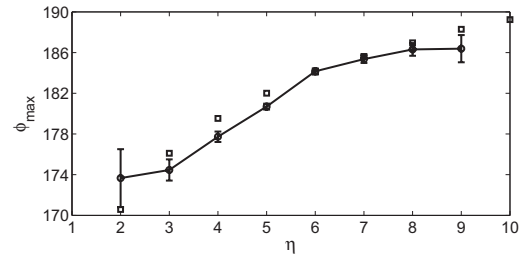


**FIGURE 4.** The average maximum angle in an arch of $\eta$ beads, as measured from the experiment (*circles with error bars*). Squares represent the expected value if $\eta$ beads are taken with angles chosen randomly from the measured angle distribution shown in Fig. 3 (c).

in the arch as having an angle chosen randomly from the observed distribution. Consequently, the larger the number of beads forming an arch, the higher the probability that a defect appears in the arch. Each bead contributes to this probability in a fixed amount. Although the underlying assumptions are not strictly proved, at least the statement is verifiable. In order to do this, we measure the largest angle of the arch, called $\phi_{max}$. The average of $\phi_{max}$ for the arches of a given geometry, *i.e.* a given number of beads $\eta$, can be obtained experimentally. Besides, the expected value of $\phi_{max}$ can also be calculated by taking $\eta$ beads randomly from the angle distribution given in Fig. 3 (c). The procedure is akin to a statistical test of hypothesis, with the use of order statistics. The result is shown in Fig. 4. As can be seen, it all works as if the statement (that each bead contributes with a constant amount to the probability of a defect appearing in the arch) is valid.

## ARCH ENDURANCE AND DEFECTS

If one follows the line of reasoning sketched in the preceding paragraphs, it is natural to take an step further and consider whether there is a relationship between the magnitude of the defect, as quantified by $\phi$, and the arch endurance.

As $R$ grows, obviously the number of beads in the arch increases. An increase in the number of beads in the arch entails a correspondingly probability that $\phi_{max}$ (the largest angle in the arch) grows bigger. If the arches break at the defects –if defects are indeed the weakest spots in the structure– then the bigger the defect, the easier should be to shatter the arch. A large $\phi$, widely exceeding 180°, means that the bead is hold by its neighbors, squeezed from the sides, and hanging dangerously. An arch having a defect like that must be rather unstable when submitted to a vibration. On the contrary, if an arch has no defects (because all the angles are smaller than 180°) no bead is hanging from above its equator. The beads are leaning against their neighbors (one of which at least must be below the bead) and the arch should endure better the vibration. In other words, if this conjecture is true, arches would be weaker not because they are larger, but because they have a higher probability of possessing bigger defects.

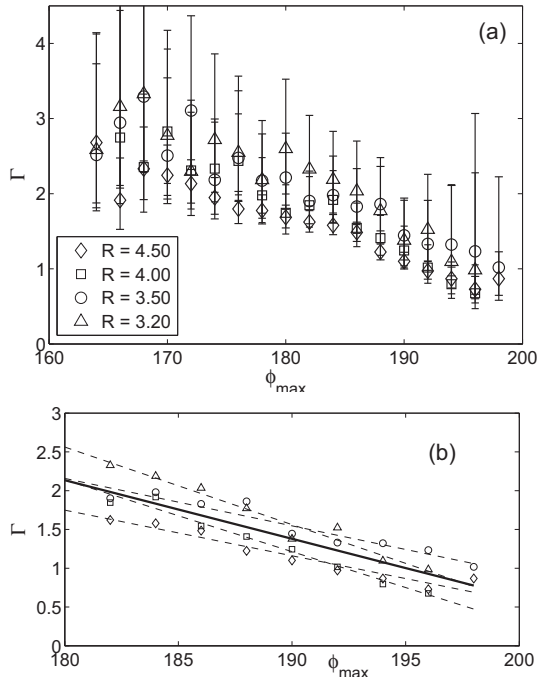We have checked this by taking the largest angle of

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**FIGURE 5.** **(a)** The acceleration $\Gamma$ at which arches break, as a function of the maximum angle in the arch $\phi_{max}$, for the different orifice sizes $R$. **(b)** A zoom of (a) in the region $\phi_{max} > 180$. Dashed lines are linear fits of each data set, the solid line is a linear fit of all data sets.

the arch, $\phi_{max}$, and measuring the acceleration needed to break it. There is a linear relationship between $\Gamma$ and $\phi_{max}$, as displayed in Fig. 5. The relationship is fulfilled for all the orifice sizes explored. An explanation for the linearity of the relationship between $\Gamma$ and $\phi_{max}$ for $\phi_{max} > 180°$ is given in [13].

Indeed, one can now travel all the way back to Fig. 1. From the orifice size $R$, one can estimate the average number of beads $\eta$ needed to build an arch to block it (Fig. 2). Knowing the angle distribution, (Fig. 3 (c)), which is general, the expected value of $\phi_{max}$ can be calculated (this is done by working out the order statistics and obtaining the expected value of the biggest angle among a set of $\eta$ beads). And from the value of $\phi_{max}$, the acceleration $\Gamma$ is found from the measurements provided in Fig. 5. The acceleration obtained from the individual data sets (*dashed lines*) are the asterisks shown in Fig. 1; the values obtained from the combined data sets (*solid line*) are represented with squares. Part of the dependence of $\Gamma$ on $R$ can then be explained by our argument; a residual dependence of $\Gamma$ on $R$ is nevertheless visible.

In summary, we have been able to show that the defects are *the weakest link* in an arch. These are the beads that hang from their neighbors at large angles. The angle

subtended between the bead center and the centers of the two adjacent beads, therefore, can be used to quantify the importance of the defects. There is a direct relationship between the largest angle in an arch ($\phi_{max}$) and the acceleration of the vibration needed to break the arch ($\Gamma$). This relationship is the best predictor for arch endurance, and can explain the observed behavior for the stability of arches against vibrations.

## ACKNOWLEDGMENTS

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