International Journal of Bifurcation and Chaos, Vol. 4, No. 5 (1994) 1113–1119

RESONANTLY FORCED SURFACE WAVES IN A NATTERER TUBE. PATTERNS, DEFECTS AND SQUEEZES

W. GONZÁLEZ-VIÑAS and J. SALÁN
Departament E.C.M., Facultat de Física, Universitat de Barcelona, Diagonal, 647 E-08028 Barcelona, Spain

Received October 14, 1993; Revised February 5, 1994

Experimental results on interfacial waves forced periodically and perpendicular to gravity which appear in a Natterer tube are presented. If the control parameter is moved, the patterns stamped by these waves will evolve with defects and more complex structures appearing, which lead to chaos. Also, squeezes (a few drops produced by squeezing) appear that move along the tube and interact with each other.

1. Introduction

There have lately been many contributions, both theoretical and experimental, studying hydrodynamical instabilities and their role in pattern formation (thermal convection, Marangoni phenomena, etc). Special interest has been devoted to the appearance of such spatiotemporal structures and their parametric evolution, and to the appearance of defects when you move away from the threshold. Into this kind of phenomena, fluid-fluid interfaces periodically excited [Fauve *et al.*, 1992; Nobili *et al.*, 1988; Funakoshi & Inoue, 1988] give dynamics of particular interest.

Without making an exhaustive review, among the theoretical approaches to this kind of problem surely the most complete analysis is that of J. W. Miles (Miles & Henderson [1990] and references therein). For interfaces periodically excited perpendicular to gravity, Miles [1984] developed a method based on an averaged Lagrangian which yields a system of ordinary differential equations for the relevant degrees of freedom. The only free parameter of Miles' theory is a damping factor heuristically added. Depending on the frequencies, amplitudes and aspect ratios (considering circular geometries) there may appear a superposition of resonant perpendicular modes, yielding chaotic regimes for applied frequencies near the natural resonance frequency.

From an experimental point of view, although there have been many works in the last 10 years about the appearance and evolution of patterns and the transition to chaos for the Faraday instability [Ciliberto & Gollub, 1984; Tufillaro *et al.*, 1989; Douady *et al.*, 1989; Ezersky & Rabinovich, 1990], there are not many experimental data for systems excited perpendicular to gravity [Nobili *et al.*, 1988; Funakoshi & Inoue 1988]. The main purpose of these is to describe experiments in which all the phase diagrams of the system have been measured and compared with the predictions of Miles [1984].

We report some experimental results about the interfacial waves, periodically and horizontally forced, that appear in a tube with CO_2 at vapor pressure (see Sec. 2 below and Fig. 1). The considered geometry, with the aspect ratio very small, diminishes the class of solutions given by Miles at linear or weakly nonlinear regimes, depending on the applied frequencies. In fact, in this paper a wide range of frequencies was explored far away from

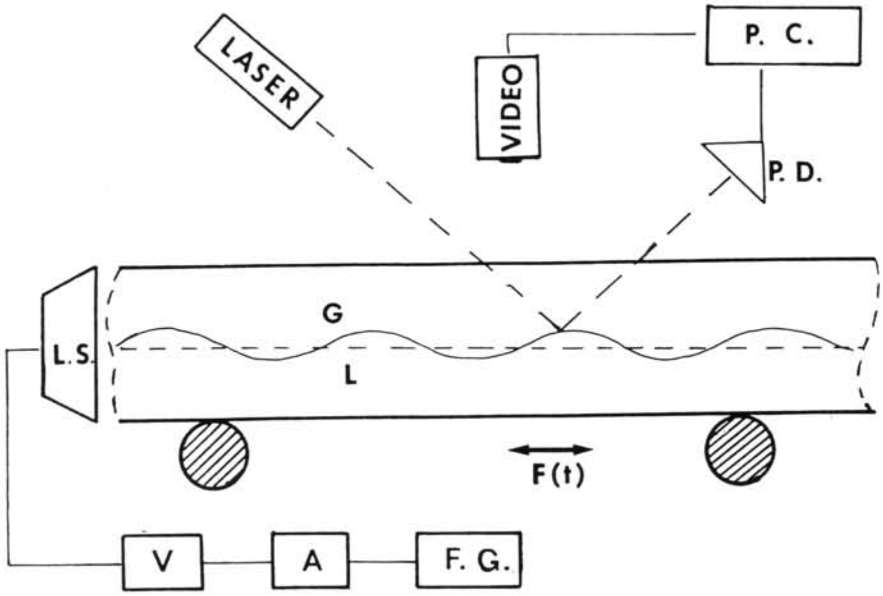

Fig. 1. Schematic sketch of the experimental set-up. LS, A, V, PD, G, L denote respectively loudspeaker, amplifier, voltmeter, photodetector, gas phase and liquid phase.

the natural resonance, reporting results in regimes where either gravity and capillary effects are negligible or both are important. Relevant points for the system considered, such as viscosity, surface tension and density, make it, besides the fact of considering a closed basin, difficult to consider the models cited above [Fauve *et al.*, 1992], and only a few well established results of the general theories of interfacial waves [Segel, 1987] are obtained. Finally, we report results obtained when, for different applied frequencies, the applied amplitudes are increased until turbulent regimes are reached, obtaining several spatiotemporal patterns. So, we show the apparition of patterns and defects, similar to those of other kinds of hydrodynamic instability [Joets & Ribotta, 1991; Pocheau 1983; Wesfreid & Zaleski, 1984]. Also, we report the apparition of drops produced by squeezing the interface (squeeze) at the extremities which move along the tube. Those squeezes play the role of particles and the patterns play the role of the fields. Similar kind of experiment on Faraday instability using CO_2 very near to the critical point has been developed by Fauve *et al.* [1992].

2. Experimental Set-Up

Experiments were performed using a Natterer tube mounted horizontally in a plateholder, minimizing lateral vibrations and mechanic frictions (Fig. 1). The Natterer tube is a CO_2 filled tube at vapor pressure for room temperature (near the critical point). In our case, the geometry was a cylindrical tube of 1 cm diameter and 28 cm length. The CO_2 liquid phase filled just one half of the whole volume of the tube; therefore, in its horizontal position the maximum depth of the liquid phase was 0.5 cm.

The physical properties are very sensitive to temperature fluctuation near the critical point (32°C). Thus we used a thermostated chamber at constant temperature and controlled externally. For the present work, the experiments were done at 20°C with a stability better than 0.2°C per hour. At that room temperature the main physical characteristics are [Weast, 1983]:

$$\begin{gathered}\rho_l = 773\frac{\mathrm{Kg}}{\mathrm{m}^3}\,, \quad \rho_g = 189\frac{\mathrm{Kg}}{\mathrm{m}^3}\,, \\ \nu_l = 9.2\cdot 10^{-8}\frac{\mathrm{m}^2}{\mathrm{s}}\,, \quad \nu_g = 7.8\cdot 10^{-8}\frac{\mathrm{m}^2}{\mathrm{s}}\,, \\ \sigma_0 = 1.16\cdot 10^{-3}\frac{\mathrm{Kg}}{\mathrm{s}^2}\,.\end{gathered} \tag{1}$$

A sinusoidal mechanical force was applied by a loudspeaker in the direction of the long axis of the tube. The driving force and amplitude were produced by a function generator connected to an amplifier with very low noise. Measurements of the driving amplitudes of the applied force were done by a microvoltmeter connected directly to the loudspeaker, because we had verified that the mechanical displacements (about 1 mm) of the tube were proportional to the applied voltages. The experiments were performed for frequencies between 10 and 100 Hz, having a resolution of 10 mHz.

The deformation of the liquid-gas interface was detected by two different methods. The first was a Schlieren method which allows us to visualize spatial structures. The second consisted in detecting variations of light intensities of a laser beam reflected from the interface. Both methods allow us to distinguish variations of the interface (with respect to the rest position) of 10 microns. Also, a system of direct visualization by a video camera CCD was used to study the patterns and their evolution when the control parameters (frequencies and amplitudes) were changed. In all the cases we synchronized the signal with the applied force to avoid stroboscopic effect.

For the working frequencies, any mechanical excitation produces waves on the borders (the viscosity is so small that the effects of the lateral walls can be neglected) which propagates to the centre of the tube.

Only when the amplitudes of these waves reach a certain value could they be visualized or detected by the methods cited above. This allows us to define a "threshold": for each applied frequency the amplitude of the driving force which gives us the first detected surface deformation is considered the "threshold" amplitude. A typical pattern of interfacial waves, observed from above, contains dark and bright stripes that correspond to crests and troughs of the waves.

The reflected laser beam mentioned above was registered by a photodetector. Deformations of the interface produced strong variations of the laser spot's surface, and then variations of the output intensities in the photodetector were related to the changes of the interface's curvature. The signal of the photodetector was digitized and processed for temporal F.F.T. analysis. This system provided an accurate method for measuring the "thresholds", and also for studying the different characteristic frequencies mixed up in the problem.

On the other hand, direct images of the patterns were acquired by a CCD camera and digitized for processing in a personal computer.

3. Results

Waves are produced periodically in the direction of the tube's long axis. There exists a predominant wave number for each driving frequency which gives the pattern: stripes perpendicular to the long axis.

We measured the "threshold" amplitudes for different driving frequencies, which had a minimum at 28 Hz, distinguishing two regions [González-Viñas & Salán, 1994a]. For frequencies lower than the minimum there was a region where the surface tension effects were not important (gravity waves). For higher frequencies there was a region where surface tension effects completely screened the effect of gravity (capillary waves). A couple of representative images is shown in Fig. 2. Measuring the "threshold," we observed hysteresis around the minimum when the driving amplitude was increased and decreased consecutively. Wave numbers for the

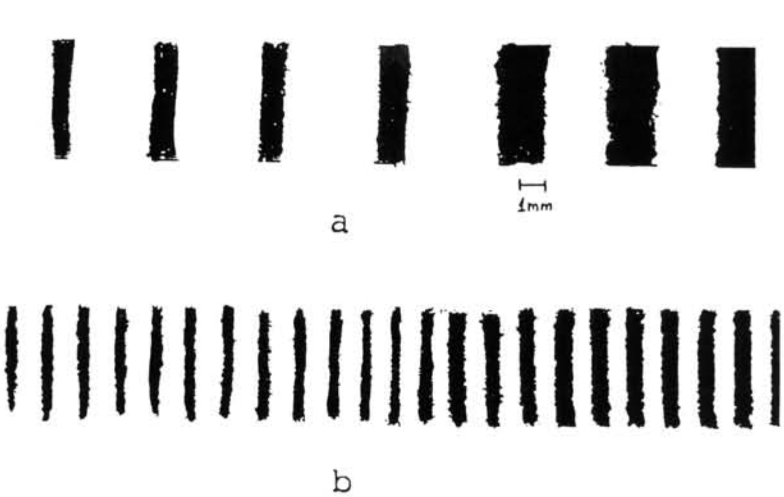

Fig. 2. Pattern for (a) gravity waves (15 Hz, dominance = 0.4) (b) capillary waves (50 Hz, dominance = 5). Dominance is the square of the wave number relative to the capillary wave number.

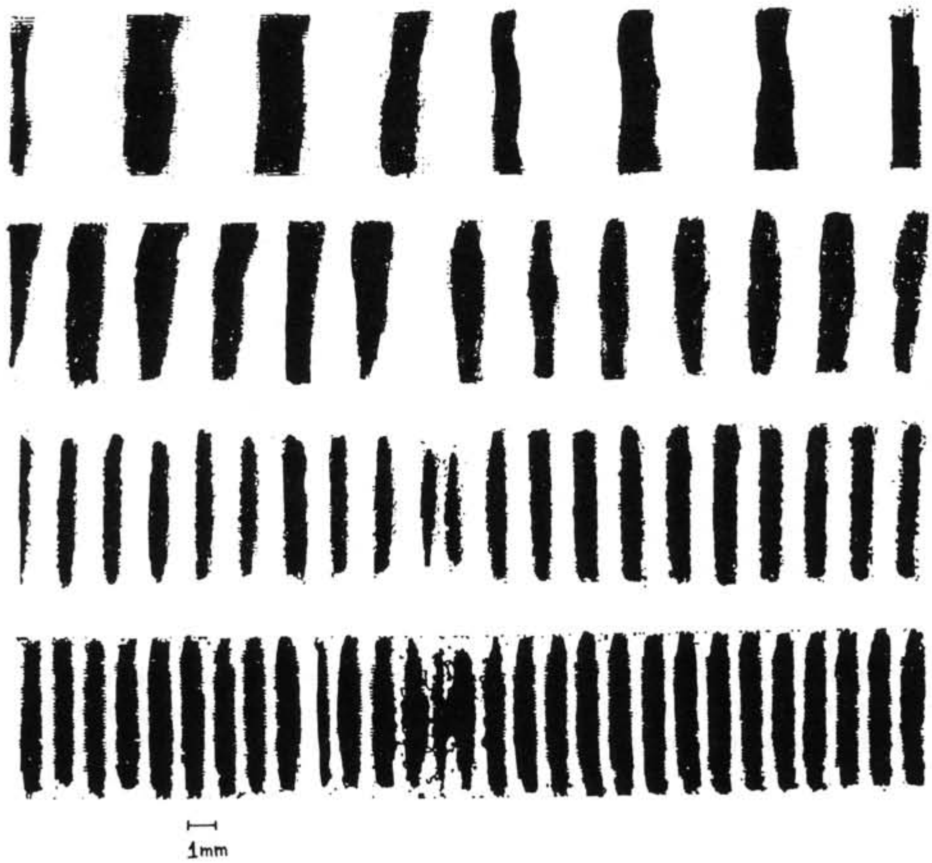

Fig. 3. From top to bottom, images with increasing driving frequencies (15, 30, 50, 80 Hz).

various applied frequencies at the "threshold" are monotonously increasing [González-Viñas & Salán, 1994a,b] (Fig. 3). If the exciting amplitude is abruptly increased from 0 to the "threshold" there will be a delay time before any pattern can be observed. Note that it is not possible to measure any delay time by slowly varying the driving amplitude. As it will be seen later, the variation of the delay time of the gravity wave region is relevant to understand the "threshold" behavior.

Propagating linear patterns appeared for frequencies higher than a cut-off frequency (22 Hz), corresponding to the capillary wave number theoretically calculated from the physical properties of the system reported above. It is easy to show that

$$\lambda_c = 2\pi \left[\frac{\sigma_0}{(\rho_l - \rho_g)g}\right]^{\frac{1}{2}} \approx 3.3 \text{ mm}$$

and that it corresponds to an adimensionalized wave number of 167 and an exciting frequency of 22 Hz. (by experimentally observing which frequency corresponds to λ_c) which differs little from the minimum amplitude's frequency (28 Hz). In Fig. 9 are shown the regions where stationary or traveling pattern exist. Their velocities were given by the slope of the stripes in a spatiotemporal acquisition (Fig. 4). For frequencies higher than 22 Hz, the velocities vary strongly up to 0.8 mm/s and afterwards smoothly up to 1.4 mm/s within the experimental frequency range. Further, we also determined whether or not there existed a modulation amplitude near the "threshold". We found that they did. Unfortunately, we are only in a position to say that the modulation amplitude frequencies were smaller than 100 mHz. In fact, the measured frequencies had fluctuations into the range of the measurements.

When the driving amplitude was increased, nonlinear effects started to become important. Perpendicular modes are unstable over some exciting amplitude. Such modes were displayed in the form of the structure's undulations (Fig. 5) with a wavelength double that of the tube's width (its existence should be placed at region d of Fig. 9). These undulations make themselves stronger and there appear

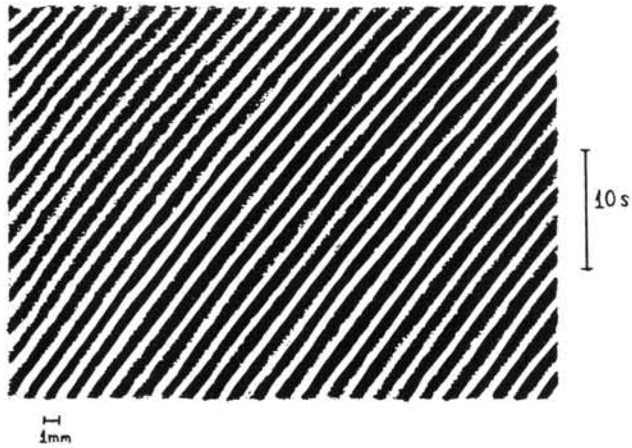

Fig. 4. One-dimensional spatiotemporal digital image acquisition showing pattern propagation at 60 Hz with a velocity of propagation near to 0.8 mm/s. Increasing time from top to bottom.

Fig. 5. Undulated structure for amplitudes higher than "threshold."

Fig. 6. A couple of defects at a driving frequency of 50 Hz.

defects that propagate along the tube. The more stable defects were a couple of defects in phase opposition (Fig. 6) that led (increasing amplitude) to more complex structures such as polygonal structures, varicose-like structures and other patterns that yields to chaos and, finally, to developed turbulence (region ct in Fig. 9). If the driving amplitude was increased, there would also appear lines of defects (Fig. 7) as a consequence of the undulation mentioned before.

Fig. 7. Formation of a defects' line (right) from an undulated structure (left).

For a given exciting frequency there exists a squeeze's threshold (dashed line in Fig. 9) for the driving amplitudes. The boundary conditions at the extremities squeeze the interface creating squeezes which move quickly and at nearly constant velocity. Squeezes usually have elastic collisions, sometimes annihilate, and rarely have inelastic collisions. When squeezes knock against the boundaries they reflect and moved in the opposite direction. If the driving amplitude is decreased the squeezes are reabsorbed, defining a drip's threshold (we called them drips for the reason that they fall) (Fig. 8). There existed hysteresis for creation and annihilation of squeezes, that is: a drip's threshold is lower than a squeeze's threshold for frequencies higher than 70 Hz (dotted line in Fig. 9).

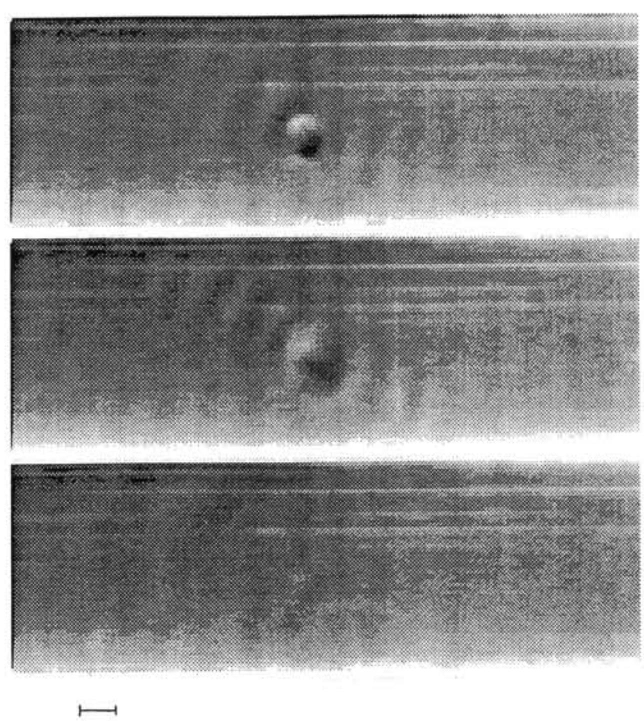

Fig. 8. Reabsorption of squeeze.

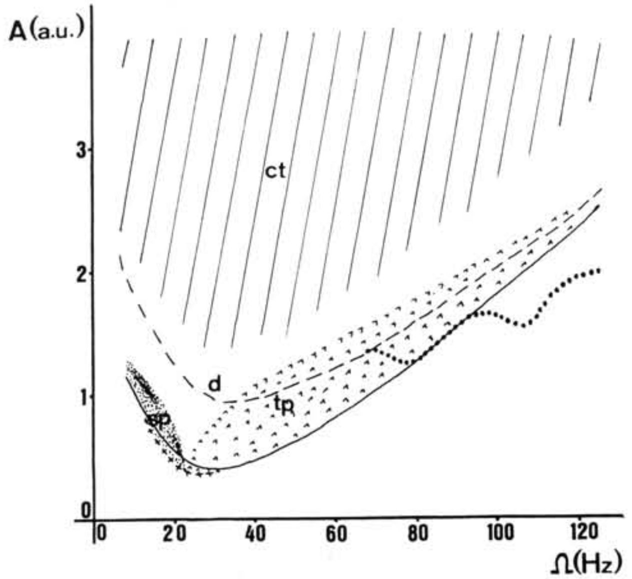

Fig. 9. Schematic parameters' plane for driving amplitudes versus driving frequencies, where the solid line represents "threshold" with increasing A for each Ω and the line formed by crosses represents the region of hysteresis at "threshold" with decreasing A for each Ω. The dashed line represents the squeezes' threshold and the dotted line the drip's threshold at hysteresis zone. Sp means stationary patterns, tp travelling patterns, d defects, and ct chaotic and turbulent motion.

4. Discussion

In this work it is revealed that, with increasing supercritically applied amplitudes for each applied frequency, transition to turbulence is made by reaching the different regions (Fig. 9). In many aspects, the observed patterns at various regimes appear different from those observed or predicted in other previous works [Tufillaro *et al.* 1989; Fauve *et al.* 1992], but, in fact, the geometry considered here is for aspect ratios where the modes parallel to the tube's axis are strongly restricted (only undulations of modes perpendicular to the axis appear for amplitudes very far from the "threshold" values).

A first interesting feature to discuss is the curve of variations of the "threshold." As it was said before, we can distinguish two regions depending on the applied frequency. The first one, for frequencies greater than 22 Hz, where surface tension effects are important, shows that the "threshold" amplitudes also grow when frequency is increased. As the wavelengths decrease with applied frequency, this kind of variation can be understood by considering simple arguments concerning interface deformation which depends mainly on the wavelengths and surface tension. On the other hand, when the driving frequency is less than 22 Hz, gravity effects predominate upon the capillary and the observed variations of the "threshold" amplitudes cannot be explained by arguments similar to the above. Here it would be necessary to consider the effects of interactions between nearest modes at the resonant curves, for each applied frequency. As the discussion would carry us out of the scope of this paper, we refer to a more detailed analysis of the problem [González-Viñas & Salán, 1994a,b]. The hysteresis around the minimum is just a nonlinear effect of the resonance generated by the equality of the velocities of neighboring modes [Segel, 1987; Hammack & Henderson, 1993]. In fact, it lies between the resonances with the first subharmonic mode and the second harmonic being a maximum hysteresis at the resonance with modes very nearly at the capillary wave number (line formed by crosses in Fig. 9).

Patterns near the "threshold" are stationary or propagative, there existing a cut-off frequency which agrees with the capillary-gravity frequency. This agreement is explained by the existence of a nonlinear parametric resonance for the surface tension, which appears as a dynamic contribution to the Laplace equation parallel to the interface [González-Viñas & Salán, 1994b]. A schematic picture (Fig. 9) shows the main features explained in the text.

As it was described in the previous section, for amplitudes far from the "threshold" value, defects appear in several ways. Two typical features are shown in Fig. 6 and Fig. 7. First, there appear undulations of the waves, and the defects that are shown in Fig. 6 are produced by spatial variation of the phase, as it was observed. Also, strong undulations produce, finally, line defects or more complicated patterns. At the present state of experiments it is difficult to discuss these results, because more systematic experiments will be necessary on this subject. In particular, it was revealed as an important problem to study the dynamics of the defects, as they travel or annihilate very quickly.

Acknowledgment

We acknowledge with thanks the partial support of M. G. Velarde. The work of one of us (W.G.V.) was supported by Comissionat per a Universitats i Recerca de la Generalitat de Catalunya under grant F.I.(93).

References

Ciliberto, S. & Gollub, J. P. [1984] "Pattern competition leads to chaos," *Phys. Rev. Lett.* **52**, 922–925.

Douady, S., Fauve, S. & Thual, O. [1989] "Oscillatory phase modulation of parametrically forced surface waves," *Europhys. Lett.* **10**, 309.

Ezersky, A. B. & Rabinovich, M. I. [1990] "Nonlinear wave competition and anisotropic spectra of spatio-temporal chaos of Faraday ripples," *Europhys. Lett.* **13**, 243–249.

Fauve, S., Kumar, K., Laroche, C, Beyseus, D. & Garrabos, Y. [1992] "Parametric instability of a liquid-vapor interface close to the critical point," *Phys. Rev. Lett.* **68**, 3160–3163.

Funakoshi, M. & Inoue, S. [1988] "Surface waves due to resonant horizontal oscillation," *J. Fluid Mech.* **192**, 219–247.

González-Viñas, W. & Salán, J. [1994a] "Surface waves periodically excited in a CO_2 tube," *Europhys. Lett.* **26**, 665–670.

González-Viñas, W. & Salán, J. [1994b] "A model for surface waves periodically excited in a CO_2 tube," submitted to *Phys. Fluids.*

Hammack, J. L. & Henderson, D. M. [1993] "Resonant interactions among surface water waves," *Annu. Rev. Fluid Mech.* **25**, 55–97.

Joets, A. & Ribotta, R. [1991] "Localized bifurcations and defect instabilities in the convection of a nematic liquid crystal," *J. Stat. Phys.* **64**, 981–1005 and references therein.

Miles, J. W. [1984] "Resonantly forced surface waves in a circular cylinder," *J. Fluid Mech.* **149**, 15–31.

Miles, J. W. & Henderson, D. M. [1990] "Parametrically forced surface waves," *Annu. Rev. Fluid Mech.* **22**, 143.

Nobili, M., Ciliberto, S., Cocciaro, B., Fetti, S. & Fronzoni, L. [1988] "Time-Dependent Surface Waves in a horizontally oscillating container," *Europhys. Lett.* **7**, 587–592.

Pocheau, A. [1983] Thèse de 3ème cycle, Université Paris-6.

Segel, L. A. [1987] Mathematics applied to continuum mechanics. (Dover Publications, New York) Part C.

Tufillaro, N. B., Ramshaukar, R. & Gollub, J. P. [1989] "Order-disorder transition in capillary ripples," *Phys. Rev. Lett.* **62**, 422–425.

Weast, R. C. (Ed.) [1983] CRC Handbook of chemistry and physics. (CRC Press).

Wesfreid, J. E. & Zaleski, S. (Editors) [1984] "Cellular structures in instabilities," in *Lecture Notes in Physics*, vol. 210 (Springer-Verlag, Berlin).