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Fractional Integration and the Dynamics of UK Unemployment

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ABSTRACT

This article is concerned with the dynamic behaviour of UK unemployment. However, instead of using traditional approaches based on $I(0)$ stationary or $I(1)$ (integrated and/or cointegrated) models, we use the fractional integration framework. In doing so, we allow for a more careful study of the low frequency dynamics underlying the series. The conclusions suggest that the UK unemployment may be explained in terms of lagged values of the real oil prices and the real interest rate, with the order of integration of unemployment ranging between 0.50 and 1. Thus, unemployment shows the characteristics of long memory but is mean reverting.

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Introduction

Although the study of unemployment behaviour has been a major preoccupation for macroeconomists and labour market economists there is a general view that it is still not well understood. Recent contributions echoing this pessimistic conclusion are found in Carruth, Hooker and Oswald (1998) in a study of US unemployment, and Bean (1994) and Nickell (1997) in their general surveys of unemployment models. The problem that has increasingly become evident even in models using a large set of labour supply and institutional factors is that they appear structurally (i.e. parameter) unstable. This problem, which has bedevilled the empirical literature is, in our view, traceable to difficulties in distinguishing between the long and short run dynamics of unemployment.

We contend that the clue to understanding unemployment lies in a more refined empirical distinction between its long and short run movement than has been the case up to now, and we illustrate this with an application to UK unemployment. This uses an alternative approach which, though motivated by a more careful treatment of the time-series characteristics of the unemployment rate, also has vitally important economic implications. It starts from the proposition that the root in the unemployment rate may be close to, but is not equal to, unity. In section 3 we describe how this may be established using fractional integration methods. This leads to the model where the reduced form for unemployment may have very considerable persistence but where its equilibrium (which we refer to as u^{**} to differentiate it from a NAIRU) is shifted by only a small number of exogenous shocks such as oil prices or real interest rates.

Models emphasising the dynamics of unemployment are found in Henry and Nixon (2000), Funke (1999), and Henry, Karanassou and Snower (1999). Among these Funke is the only study to use fractional integration methods, although unlike the present one the model he uses is a univariate one. The contribution which the present paper makes is to extend models of this latter sort by investigating the case for low frequency dynamics in a reduced form model of unemployment using fractional integration methods.

The rest of the paper sets out some of the claims made so far in a more formal way, and some of the technical issues involved. Section 3, sets out the details on the estimation procedure, and Section 4 presents the application to UK unemployment. Section 5 concludes.

Modelling unemployment

As already mentioned we focus on reduced-form dynamic models of the unemployment rate in what follows. To motivate our approach to this problem we begin with a familiar model. This is the example reported in Bean and Layard (1988) who estimate unemployment persistence using a second-order autoregressive equation for u_t based upon a model of long-term unemployment and insider effects on wages and prices. Their unemployment equation takes the form

$$(1 - (1 - s - ch)L - (a + s(1 - c)h)L^2)u_t = bx_{t-1} \quad (1)$$

where L is the backward shift operator and the parameters are defined next. These parameters are then the inflow into unemployment from employment which is given by s , the parameter c which indexes the effectiveness of workers when exiting unemployment, where the longer they remain unemployed, the lower is their effectiveness, and h is the slope of the hiring function. The right hand side of (1) is a set of shift variables (x). In their example, Bean and Layard (1988) take these to include both supply and demand shocks. That aside, the emphasis in their paper, and ours, is on the order of the dynamics in the unemployment equation given by the polynomial in L , which determines the response of unemployment to changes in the variables in the vector Z . Clearly, in this example, the degree of persistence in unemployment depends upon how quickly workers flow into and out of unemployment, and the strength of insider effects. Henry and Nixon (op.cit) discuss how such a model can be extended to allow for hiring and firing costs and employment dynamics and capital constraints on employment, leading to a higher order auto-regression in unemployment. In such higher-order cases, however, it is not possible to relate the parameters in the auto regression to the underlying structural parameters of the theoretical model.

Starting from this basic framework of a single dynamic equation linking unemployment to shift variables (which are implicitly weakly exogenous), we address two main points. First are the conditions under which the unemployment equation can be treated as a reduced form met in this case?. Second what are the advantages in modelling the equation dynamics using fractional, as opposed to non-fractional integration, processes. We discuss these two points briefly next.

A Reduced Form Equation for Unemployment.

To treat (1) as a reduced form equation, it is necessary that the set of x variables in the equation may be treated as a valid conditioning vector only if they are weakly exogenous to the parameters of interest in that equation. In other words weak exogeneity requires that where equation (1) is the conditional process the marginal process driving the x variables does not contain information, which is relevant for valid inferences to be made about the parameters of interest in the model. (see Banerjee, Dolado, Galbraith and Hendry (1993) for a full account). To establish whether the parameters of the conditional and the marginal model we use later are variation free or not, we undertook some simple tests. These use only the real oil price and real interest rates in the x vector in equation (1), as this is the variable set we use later. (Section 4 below gives details of the data used throughout the rest of the paper). The tests involved tests of weak and strong exogeneity, where to establish evidence for Granger noncausality, an unrestricted VAR of the three variables in the model; the unemployment variable (u), real oil prices and the real interest rate was used. A fourth order VAR was found to be sufficient for this. Weak exogeneity appeared to be satisfied in dynamic equations for the real interest and oil price, since when entering the current value of u in these equations it proved to be insignificant. (Having t statistics of 1.2 and 0.9 respectively). This finding supports the view that the real interest rate and the real oil price are weakly exogenous for the model. Exclusion tests conducted in the fourth-order VAR then confirmed that lagged values of the unemployment variable could be excluded from the real interest rate and real oil price equation (The LM (4) and LR (4) tests were 5.2 and 5.4 for the real interest equation, and 4.1 and 4.2 for the real oil price

equation respectively) which, coupled with the finding of weak exogeneity, implies that the oil price and interest rate can be treated as strongly exogenous in this application.

Fractional Integration

Formally, the fractionally integrated structure imposes a slow rate of decay on the autocorrelations (in fact, these are given by a hyperbolic weighting function), unlike standard autoregressions where the decay is exponential and usually fairly rapid. Furthermore, this way of specifying the model, allows us to consider both I(1) and the I(0) specifications as particular cases of the more general class of model, the I(d) class, where d can be any real number.

Before proceeding with the fractional model, we report a preliminary exercise done on with standard tests of integration of the relevant variables. This preliminary is necessary to ensure that u and the x variables are integrated of the same order so that estimates of persistence obtained from the reduced form model (given in the fractional form by equation (3) below) does not suffer from misspecified dynamics due to different orders of integration in the variables. The table below reports on the orders of integration of the variables used.

	ADF	ADF(4)
Real interest rate	-2.12	-3.26
Real oil price	-1.67	-1.5
Terms of trade	-1.98	-2.27
Unemployment	-0.74	-2.02

(Full definitions of the variables are given in Section 4).

Prima-facie these results are consistent with the contending variables used in the later fractional tests being of similar orders of integration. As the cut-off value for the ADF test is -3.44 , these tests are consistent with the variables being I(1). Next, there is clear evidence that the unemployment and the x variables cointegrate. Johansen maximum likelihood tests each confirm this. The Likelihood Ratio test for at least one cointegrating vector is 43.2 (31.8), and the Trace test is 72.6(63.0), with the VAR with lag length of 4. (Figures in parenthesis are the 95% critical values).

A word of caution is necessary here. It is well known that these standard tests have low power where the alternative is a root close to -1 but not equal to -1 , which is precisely the case which is the focus here. Indeed, we illustrate this when we test the unemployment series for evidence of fractional integration in Section 4 later. Nevertheless, such a standard test is a necessary preliminary to these later tests, as they provide some evidence that the dynamics which we identify later are not likely to be due to the contending variables being of different orders of integration and, as a result, contaminating our estimates of the dynamics in the unemployment series.

In order to bring both ordinary and fractionally integrated formulations together, consider a reduced form equation for the unemployment rate (u_t),

$$\Phi(L) u_t = \beta_1' \Theta(L) x_t + \varepsilon_t, \quad (2)$$

where ε_t is a white noise process. To describe the extension of the fractionally integrated model, suppose we have

$$u_t = \beta_1' \Theta(L) x_t + v_t, \quad (3)$$

$$\Phi^*(L) (1-L)^d v_t = w_t, \quad (4)$$

with w_t being $I(0)$, and where d is a given real number and $(1-L)^d$ is expressed in terms of its Binomial expansion

$$(1-L)^d = \sum_{j=0}^{\infty} \binom{d}{j} (-1)^j L^j,$$

such that

$$(1-L)^d v_t = v_t - d v_{t-1} + \frac{d(d-1)}{2} v_{t-2} - \frac{d(d-1)(d-2)}{6} v_{t-3} + \dots$$

Substituting (4) into (3), we obtain

$$\Phi^*(L) (1-L)^d u_t = \beta_1' w(L) x_t + w_t,$$

where $w(L) = \Phi^*(L) \Theta(L) (1-L)^d$. Then, if $d = 0$, u_t is an $I(0)$ stationary process and if $d = 1$, we have an integrated model for unemployment; the two models already described. But as d can be any real number, this permits a richer characterisation of the dynamics affecting unemployment compared with that imposed by the $I(1)$ and $I(0)$ specifications. Furthermore, if d in (4) belongs in the interval $(0, 0.5)$, the series is still covariance stationary but the autocorrelations take far longer to decay to zero than those based on $d = 0$. In addition, if $d \in (0.5, 1)$, the process is no longer stationary but still will be mean-reverting, with shocks affecting the series but this returns to its original level sometime in the future.

Thus, d plays a crucial role in explaining the degree of persistence of the series. Processes like (4) with positive non-integer d (and $\Phi^*(L) = 1$) are called fractionally integrated and when w_t is $ARMA(p, q)$, v_t is a fractionally $ARIMA$ ($ARFIMA(p, d, q)$) process. This type of model was introduced by Granger and Joyeux (1980), Granger (1980, 1981) and Hosking (1981) and were justified theoretically in terms of aggregation by Robinson (1978) and Granger (1980), and more recently, in terms of the duration of shocks by Parke (1999). In the following section, we use a testing procedure suggested by Robinson (1994) for testing the applicability of models of this type to the UK unemployment rate.

The testing procedure for fractional integration

To define the testing procedure for d , consider the model given by a simplification of (3) and (4) above, namely

$$u_t = \beta' x_t + v_t, \quad (5)$$

$$(1 - L)^d v_t = w_t, \quad (6)$$

where u_t is the observed dependent variable and x_t is a $(k \times 1)$ vector of weakly exogenous variables. The error term w_t is an $I(0)$ process with parametric spectral density f , which is a given function of frequency λ and of unknown parameters,

$$f(\lambda; \sigma^2; \tau) = \frac{\sigma^2}{2\pi} g(\lambda; \tau) \quad -\pi < \lambda \leq \pi,$$

where the scalar σ^2 and the $(q \times 1)$ vector τ are unknown but the function g is assumed to be known. For example, in the AR case, if $\sigma^2 = V(\varepsilon_t)$, we have

$$g(\lambda; \tau) = |\Phi(e^{i\lambda})|^{-2},$$

where Φ is the AR polynomial, so that the AR coefficients are functions of τ .

Robinson (1994) defines tests of the null hypothesis:

$$H_o : d = d_o \quad (7)$$

for any given real number d_o in (5) – (7). To derive the test statistic, we form

$$z_t = (1 - L)^{d_o} x_t; \quad x_t = 0, \quad t \leq 0.$$

Based on the null differenced model, the least-squares estimate of β and residuals are

$$\hat{\beta} = \left(\sum_{t=1}^T z_t z_t' \right)^{-1} \sum_{t=1}^T z_t (1 - L)^{d_o} u_t; \quad \hat{w}_t = (1 - L)^{d_o} u_t - \hat{\beta}' z_t, \quad t = 1, 2, \dots$$

Unless g is a completely known function (e.g., $g \equiv 1$, so that w_t is white noise), we have to estimate the nuisance parameter vector τ . One such estimate, which fits naturally into our frequency domain setting, is

$$\hat{\tau} = \arg \min_{\tau} \sigma^2(\tau)$$

where the minimisation is carried out over a suitable subset of \mathbb{R}^q , and $\sigma^2(\tau) = 2\pi \mathbf{1}'\mathbf{h}(\tau)/T$, where $\mathbf{h}(\tau)$ is the $(T-1)$ -dimensional column vector with j^{th} -element $P(\lambda_j)/g(\lambda_j; \tau)$; $P(\lambda)$ is the periodogram of \hat{w}_t ; $\mathbf{1}$ is the $(T-1) \times 1$ vector of 1's and $\lambda_j = 2\pi j/T$. Next, \hat{a} and \hat{b} are given by

$$\hat{a} = -\frac{2\pi}{T} m' h(\hat{\tau}), \quad \hat{b} = \frac{2}{T} \{m'm - m'M(M'M)^{-1}M'm\},$$

in which m is the $(T-1) \times 1$ vector with j^{th} -element given by $\log |2 \sin(\lambda_j/2)|$, and M is the $(T-1) \times q$ matrix with j^{th} -row $(\partial/\partial\tau) \log g(\lambda_j; \hat{\tau})$. Next, we write

$$\hat{s} = \left(\frac{T}{\hat{b}}\right)^{1/2} \frac{\hat{a}}{\hat{\sigma}^2}, \quad (8)$$

where $\hat{\sigma}^2 = \sigma^2(\hat{\tau})$. Under the null hypothesis (7), Robinson (1994) established under regularity conditions that

$$\hat{s} \rightarrow_D N(0,1) \quad \text{as } T \rightarrow \infty. \quad (9)$$

The conditions on w_t in (8) are far more general than Gaussianity, with a moment condition only of order 2 required. From these it follows that an approximate one-sided $100\alpha\%$ -level test of (7) against the alternatives $H_1: d > d_0$ is given by the rule ‘‘Reject H_0 if $\hat{s} > z_\alpha$,’’ where the probability that a standard normal variate exceeds z_α is α . Conversely, an approximate one-sided $100\alpha\%$ -level test of (7) against alternatives $H_1: d > d_0$ is given by the rule ‘‘Reject H_0 if $\hat{s} < -z_\alpha$.’’

As these rules indicate, we are in a classical large sample testing situation for reasons described by Robinson (1994), who also showed that the above tests are efficient in the Pitman sense that against local alternatives: $H_1: d = d_0 + \delta T^{-1/2}$ for $\delta \neq 0$, \hat{s} has an asymptotic normal distribution with variance 1 and mean which cannot (when w_t is Gaussian) be exceeded in absolute value by that of any rival regular statistic. Empirical applications based on this testing procedure can be found in Gil-Alana and Robinson (1997) and Gil-Alana (2000) and other versions of the tests based on seasonal (quarterly and monthly) and cyclical data are respectively Gil-Alana and Robinson (2001) and Gil-Alana (1999, 2001).

A notable feature of Robinson’s (1994) tests is that the null $N(0, 1)$ distribution of \hat{s} holds across a broad class of exogenous regressors x_t , unlike most of unit root tests embedded in AR alternatives, where the null limit distribution can vary with features of the regressors. In the following section we use this framework to test (7) in a model given by (3) and (4).

The empirical application: UK Unemployment

In this section the testing procedure described earlier is used to identify the dynamics of the UK unemployment rate. Firstly, we investigate its univariate behaviour, estimating

and testing its order of integration. We find there is evidence that it is fractionally integrated, so exhibits extensive persistence when shocked. Next, we investigate what the major source of these shocks are. To do this, a set of weakly exogenous regressors are included in the dynamic model of unemployment. In other words, we estimate a model based on (3) and (4) for different values of d .

The unemployment series used in this paper is the logistic transformation of the unemployment rate in the UK¹. The rate is for female and male unemployment and is from ETAS from 1971Q1. Prior to this date adjusted data from the DEG is used. The Real Oil Price is the US Producer Price Index for Crude Oil (US Survey of Current Business, Dept of Commerce), converted to Sterling using the spot exchange rate, and deflated by the GDP deflator (IMF National Accounts). The real interest rate is the Treasury bill rate less the inflation rate of the GDP deflator. Finally the Terms of Trade is defined as $s \ln(P_m / P^*)$ where s is the import share, P_m is the import price index, and P^* the world price of manufacturing exports (in Sterling)(..). The data used are quarterly and the sample size is 1966q1 - 1997q4.

A univariate model

First, to investigate the univariate properties of unemployment, as already anticipated we model this as an ARFIMA(p, d, q) model, with p and q each taking values up to and equal to 3. That is, u_t is modelled as

$$\phi_p(L) (1 - L)^d u_t = \theta_q(L) \varepsilon_t,$$

where $\phi_p(L)$ and $\theta_q(L)$ ($p, q \leq 3$) are respectively the AR and MA polynomials. Two approaches to estimating and testing this are implemented in what follows: the first a ML procedure and the second the testing procedure suggested by Robinson described in the previous section. Table 1 summarises the estimated values of d (and of the remaining parameters) when the ML procedure is used for alternative values of p and q . This estimation uses Sowell's (1992) procedure of estimating by maximum likelihood in the time domain. The results clearly indicate that practically all the estimated values of d are higher than 1. However, in eleven out of the sixteen models, the unit root hypothesis ($d = 1$) cannot be rejected. On the other hand, the null $d = 0$ is rejected in all cases. The Akaike and Schwarz information criteria both indicate that the best model specification might be an ARFIMA(0, 1.18, 3), so the unit root null hypothesis cannot be rejected in this case.

Table 1

Maximum likelihood estimation of ARFIMA(p, d, q) models for unemployment											
		t-tests ¹		AR parameters			MA parameters			Criteria	
ARMA	d	$t_{d=0}$	$t_{d=1}$	ϕ_1	ϕ_2	ϕ_3	θ_1	θ_2	θ_3	AIC	SIC
(0, 0)	1.92	21.3	10.2	--	--	--	--	--	--	221.6	220.1

¹ See Wallis (1987) for a justification based on the logistic transformation being defined between $\pm \infty$ so that standard distributions apply.

(1, 0)	1.19	7.43	1.18'	0.70	--	--	--	--	--	225.7	222.8
(0, 1)	1.74	13.3	5.69	--	--	--	0.19	--	--	221.7	218.9
(1, 1)	1.15	5.00	0.65'	0.71	--	--	0.03	--	--	224.7	220.4
(2, 0)	0.60	6.00	0.66'	1.60	-0.63	--	--	--	--	225.3	221.0
(0, 2)	1.50	12.5	4.16	--	--	--	0.33	0.44	--	226.1	221.8
(2, 1)	1.25	6.25	1.25'	0.03	0.44	--	0.59	--	--	223.9	218.2
(1, 2)	1.14	7.12	0.87'	0.71	--	--	-3013	-8764	--	223.7	218.0
(2, 2)	1.14	2.85	0.35'	0.72	-0.23	--	0.01	0.50	--	228.0	220.9
(3, 0)	0.98	3.76	-0.07'	0.89	0.06	-0.21	--	--	--	226.8	221.1
(0, 3) (*)	1.18	10.7	1.63'	--	--	--	0.77	1.65	0.92	230.3	224.6
(3, 1)	1.02	3.09	0.06'	0.92	0.02	-0.21	-0.06	--	--	225.8	218.7
(3, 2)	0.97	3.12	-0.09'	0.85	-0.51	0.28	0.05	1.39	--	228.3	219.7
(1, 3)	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----
(2, 3)	1.86	2.38	1.10'	0.92	-0.23	--	-0.85	2.08	-2.12	227.1	218.6
(3, 3)	1.88	4.70	2.20	0.41	-0.06	0.18	0.67	1.56	0.63	227.8	217.8

-----: Convergence was not achieved after 240 iterations. ' : Non-rejection values of the null hypothesis: $d = 1$ at the 95% significance level. *: Best model specification according to the AIC and SIC criteria.

Turning now to the Robinson procedure, we take the model given by (4) and (5), where $\beta = 0$ (i.e. the model is univariate). A range of different forms for w_t are tried, including where it is pure white noise, autoregressive (AR(1), AR(2)) and seasonal autoregressive (AR(1) and AR(2)). Higher order autoregressions were also performed obtaining similar results.

Table 2, then gives the estimated orders of integration of unemployment according to Robinson's (1994) tests. Using the test of d given by (6) for $d_0 = 0.00, \dots (0.25) \dots 2.00$, we observe that the null hypothesis $d = 1$ is never rejected, though we also observe several non-rejection values when $d = 0.75$ and 1.25 .

The conclusion of both of these univariate procedures applied to unemployment is that the unit root null hypothesis cannot be rejected when modelling unemployment alone. However, this feature may not be robust to extensions in the model, particularly when the likely determinants of unemployment are used in a multivariate model. We turn to consider extensions to the univariate model next.

Table 2

Testing the order of integration of unemployment with the tests of Robinson (1994)									
u_t	Values of d_0								
	0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
White noise	26.13	20.12	12.70	5.24	0.28'	-2.33	-3.72	-4.51	-5.00
AR(1)	8.98	8.97	6.18	3.27	0.59'	-1.86'	-1.98	-2.02	-2.59

AR(2)	6.39	2.39	2.31	1.68'	-0.91'	-1.61'	-2.24	-3.09	-3.99
Seasonal AR(1)	11.87	6.58	3.83	1.81'	-0.06'	-2.37	-3.75	-4.52	-5.03
Seasonal AR(2)	8.58	4.22	2.51	1.15'	-0.14'	-2.46	-3.77	-4.52	-5.03

': Non-rejection values of the null hypothesis at the 95% significant level.

A multivariate model

Modelling unemployment as a reduced form, we focus on a set of contending explanatory variables, which include labour supply variables (like the union density), real interest rates, the terms of trade and real oil prices. There are two reasons for selecting this specification. First, in a series of econometric tests Henry and Nixon (2000), find that a model based on this restricted set of driving variables is preferred to a commonly used model based on a much wider set. Second, the emphasis on real oil prices and the real interest rate enables us to compare our findings with those proposed by Phelps (1994), Carruth et al (op.cit) and Blanchard (1999), each of which has recently placed emphasis on at least one of these variables as a primary determinant of unemployment.

The sample for our model runs from 1966q1 to 1997q4. We initially employ the model given by equations (5) and (6), testing (7) for $d_0 = 0; 0.10; 0.20; \dots(0.10) \dots 0.90$ and 1.00, with x_t being the set of weakly exogenous variables just defined. Initially up to five lags is allowed in each variable. So,

$$u_t = \alpha + \sum_{k=0}^5 \beta_k r_{t-k} + \sum_{k=0}^5 \gamma_k p_{t-k} + \sum_{k=0}^5 \delta_k t_{t-k} + \sum_{k=0}^5 \lambda_k s_{t-k} + v_t \quad (10)$$

$$(1 - L)^d v_t = w_t, \quad t = 1, 2, \dots \quad (11)$$

where r_t is the real interest rate; p_t is real oil prices; t_t the terms of trade and s_t corresponds to union density.

We first computed the one-sided statistic \hat{s} given by (8) above in the model (10) and (11) when w_t is either assumed to be white noise or an autoregressive process of orders 1 or 2. Higher order autoregressions were also performed obtaining similar results to those in the AR(1) and AR(2) cases. A striking feature observed there was the lack of monotonic decrease in the value of \hat{s} with respect to d_0 . Such monotonicity is a characteristic of any reasonable statistic, given correct specification and adequate sample size. For example, if $d = 0.50$ is rejected against $d > 0.50$, an even more significant result in this direction could be expected when $d = 0.40$ or $d = 0.30$ is tested. We interpret this lack of monotonicity as reflecting possible misspecification of the model due to the inclusion of non-significant variables. So, in Table 3 we report the same statistic but only including those regressors that were significant across all the different values of d_0 . Monotonicity is not necessarily expected in this case, since the elements of x_t differ between the equations.

Table 3

Testing (7) in model given by (10) and (11) including only significant regressors for each				
d_0	Regressors (*)	White noise w_t	AR(1) w_t	AR(2) w_t
0.00	$p_{t-5}; s_t$	13.08	0.27'	-5.76
0.10	$p_{t-5}; s_t$	12.44	-0.54'	-4.31
0.20	$p_{t-5}; s_t$	12.19	-1.09'	-3.74
0.30	$p_{t-5}; s_t$	12.30	-1.99	-3.59
0.40	$p_{t-5}; (t_t); r_t$	12.62	-2.01	-2.91
0.40	$p_{t-5}; r_t$	12.59	-1.98	-3.55
0.50	$p_{t-5}; (t_t); r_t$	13.00	-2.14	-2.90
0.50	$p_{t-5}; r_t$	13.09	-1.97	-2.98
0.60	$p_{t-5}; r_t$	13.57	-2.02	-2.13
0.70	$p_{t-5}; r_t$	12.82	-1.07'	-0.08'
0.80	$t_{t-3}; p_{t-5}; r_t$	11.63	-0.60'	0.29'
0.90	$t_{t-3}; p_{t-5}; r_t$	10.76	-0.70'	0.73'
1.00	(r_t)	9.58	-2.10	-4.78
1.00	---	9.49	-1.90'	-4.05

* : In parenthesis, the non-significant regressors. ' : Non-rejection values of the null hypothesis at the 95% significance level.

The results show that when modelling w_t as white noise the null hypothesis given by (7) always results in a rejection across the different values of d_0 . But allowing w_t to follow an autoregressive process, the results differ. Thus, if w_t is AR(1), the null cannot be rejected when $d = 0, 0.10$ and 0.20 with the lagged real oil price (p_{t-5}) and the union density (s_t) as significant regressors. Also, H_0 (7) cannot be rejected when $d = 0.70, 0.80$ and 0.90 with $x_t = p_{t-5}; r_t$ (real interest rate) and t_{t-3} (lagged terms of trade) and AR(1) or AR(2) disturbances, and finally when $d = 1$ and w_t is AR(1) and do not include regressors in (10).

Table 4 summarises the selected models according to results shown in Table 3. That is, we write the estimated models based on (10) and (11), in which the null hypothesis (7) was not rejected and all the coefficients were significantly different from zero. We see that Models 1, 2 and 3 are consistent with stationary unemployment. In such situations, the real oil prices lagged five periods, along with the union density appear as significant regressors, and the coefficients are rather similar in the three models. For all the other specifications, d is greater than 0.60, indicating that unemployment may be a nonstationary series. We also see that lagged oil prices appears as a significant regressor in practically all the models, (in fact, in all except when $d = 1.00$). Surprisingly, we also observe across these models that the higher the order of integration d is, the lower the coefficient on oil prices. Thus, for example, setting $d = 0$ (in Model 1), the coefficient for p_{t-5} is 0.96; setting $d = 0.30$ (in Model 3), it becomes 0.78; and setting $d = 0.9$ (in Model 6) the coefficient reduces to 0.07. This may indicate that there may exist some kind of competition between the lagged real oil prices and the order of integration in describing the UK unemployment behaviour.

Table 4

Selected models for unemployment according to Table 1		Diagnostic* Tests
1.	$u_t = 2.49 + 0.96 p_{t-5} - 0.07 s_t + v_t$ <p style="text-align: center;">(0.28) (0.04) (0.005)</p> $v_t = 0.79 v_{t-1} + \varepsilon_t$ <p style="text-align: center;">(0.04)</p>	A; C
2.	$u_t = 2.07 + 0.87 p_{t-5} - 0.07 s_t + v_t$ <p style="text-align: center;">(0.32) (0.04) (0.006)</p> $(1 - L)^{0.10} v_t = w_t; \quad w_t = 0.76 w_{t-1} + \varepsilon_t$ <p style="text-align: center;">(0.04)</p>	A; B; C
3.	$u_t = 1.53 + 0.78 p_{t-5} - 0.06 s_t + v_t$ <p style="text-align: center;">(0.37) (0.05) (0.007)</p> $(1 - L)^{0.20} v_t = w_t; \quad w_t = 0.78 w_{t-1} + \varepsilon_t$ <p style="text-align: center;">(0.06)</p>	A; B; C
4.a	$u_t = -2.62 + 0.16 p_{t-5} - 0.019 r_t + v_t$ <p style="text-align: center;">(0.21) (0.04) (0.008)</p> $(1 - L)^{0.70} v_t = w_t; \quad w_t = 0.75 w_{t-1} + \varepsilon_t$ <p style="text-align: center;">(0.05)</p>	A; B; C; D
4.b	$(1 - L)^{0.70} v_t = w_t; \quad w_t = 0.78 w_{t-1} + 0.14 w_{t-2} + \varepsilon_t$ <p style="text-align: center;">(0.08) (0.08)</p>	A; B; D
5.a	$u_t = -2.52 + 0.11 p_{t-5} - 0.019 r_t + 0.83 tt_{t-3} + v_t$ <p style="text-align: center;">(0.27) (0.03) (0.007) (0.36)</p> $(1 - L)^{0.80} v_t = w_t; \quad w_t = 0.73 w_{t-1} + \varepsilon_t$ <p style="text-align: center;">(0.06)</p>	A; B; D
5.b	$(1 - L)^{0.80} v_t = w_t; \quad w_t = 0.76 w_{t-1} + 0.03 w_{t-2} + \varepsilon_t$ <p style="text-align: center;">(0.05) (0.02)</p>	A; B; D
6.a	$u_t = -1.66 + 0.07 p_{t-5} - 0.017 r_t + 0.73 tt_{t-3} + v_t$ <p style="text-align: center;">(0.49) (0.03) (0.006) (0.32)</p> $(1 - L)^{0.90} v_t = w_t; \quad w_t = 0.72 w_{t-1} + \varepsilon_t$ <p style="text-align: center;">(0.06)</p>	A; B; D
6.b	$(1 - L)^{0.90} v_t = w_t; \quad w_t = 0.74 w_{t-1} + 0.44 w_{t-2} + \varepsilon_t$ <p style="text-align: center;">(0.05) (0.14)</p>	A; B; D
7.a	$u_t = -1.33 + v_t$ <p style="text-align: center;">(0.06)</p>	
7.b	$(1 - L) v_t = w_t; \quad w_t = 0.82 w_{t-1} + \varepsilon_t$ <p style="text-align: center;">(0.05)</p>	A; B; D

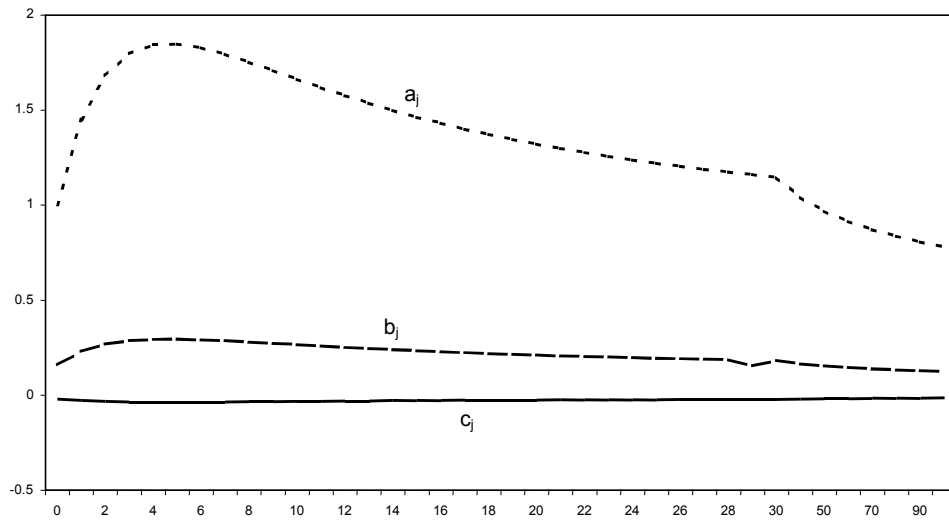
*: Non-rejections at the 99% significance level of A): No Serial Correlation; B): Functional form; C): Normality; and D): Homoscedasticity. Standard errors in parenthesis.

A more difficult task is to determine which is the correct model specification across the different models presented in that table. We display in the last column of Table 4 several diagnostic tests carried out on the residuals. In particular, we perform tests of no serial correlation, functional form, normality and homoscedasticity using Microfit. We observe that if we assume that unemployment is $I(0)$, the model fails in relation to tests of functional form and homoscedasticity. However, allowing d to have a low degree of long memory, (with $d = 0.1$ or 0.2), the models fail then only in relation to the homoscedasticity property. On the other hand, assuming nonstationarity for unemployment, (in Models 4 –7), the real interest rate becomes a significant regressor along with the terms of trade in some cases. We observe across these models that the only one which passes all the diagnostic tests on the residuals seems to be Model 4a, where unemployment is modelled as

$$\begin{aligned}
 u_t &= \alpha + \gamma_5 p_{t-5} + \beta_1 r_t + v_t \\
 (12) \\
 (1 - L)^{0.70} v_t &= w_t; \quad w_t = \phi w_{t-1} + \varepsilon_t,
 \end{aligned}$$

giving the estimates: $\alpha = -2.62$; $\gamma_5 = 0.16$; $\beta_1 = -0.019$; and $\phi = 0.75$. Thus, the impact of oil prices and interest rates is quite slow, with the adjustment process modelled through both the fractional parameter and the autoregressive coefficient.

Chart 1: Impulse response function and impacts of real oil prices and real interest rates on unemployment



a_j corresponds to the impulse response function based on the polynomial: $(1 - 0.75L)(1 - L)^{0.70}$;

b_j and c_j represents respectively the impacts of oil prices and real interest rates on the unemployment

To evaluate the responses of unemployment to a shock, we need to derive the impulse response functions. We do this next. Let $(1 - 0.75L)(1 - L)^{0.70} = a^*(L)$, and calling $k^*(L) = -2.62 a^*(L)$; $b^*(L) = 0.16 a^*(L)$; and $c^*(L) = -0.019 a^*(L)$, the model in (12) becomes

$$a^*(L)u_t = k^*(L)1 + b^*(L)p_{t-5} + c^*(L)r_t + \varepsilon_t,$$

or alternatively,

$$u_t = k(L)1 + b(L)p_{t-5} + c(L)r_t + a(L)\varepsilon_t$$

where $k(L)=k^*(L)/a^*(L)$; $b(L)=b^*(L)/a^*(L)$; $c(L)=c^*(L)/a^*(L)$ and $a(L)=1/a^*(L)$, and using now a power expansion of $a(L)$, $b(L)$ and $c(L)$ in terms of its lags, with $u_j = p_j = r_j = 0$ for $j \leq 0$, we obtain

$$u_t = k(L)1 + \sum_{j=1}^{t-1} a_j u_{t-j} + \sum_{j=0}^{t-6} b_j p_{t-5-j} + \sum_{j=0}^{t-1} c_j r_{t-j} + \sum_{j=0}^{t-1} a_j \varepsilon_{t-j}, \quad (13)$$

where a_j are the coefficients of the impulse response function, and b_j and c_j represents respectively the impacts of the real oil prices and the real interest rates on the unemployment.

Chart 1 summarizes these values for $j = 1, 2, \dots, 30, 40, 50, \dots, 100$. We observe through the a_j 's that the effect of a shock on unemployment tends to die away in the long run though it takes a very long period to disappear completely. In fact, we see that even 30 periods after the initial shock, its complete effect still remains on the series and is only after around 50 periods that it becomes smaller than 1. The impact of real oil prices is around 16% five periods later; it increases up to around 30% in the following five periods, and then starts decreasing slowly. Similarly, the current impact of the interest rate is around -1.9% and then increases up to -3.5% before falling.

We can conclude by saying that the lagged values of the real oil prices in all models and the real interest rate in some of the models play an important role in explaining the movements in the UK unemployment. They have an immediate effect but this is coupled with an adjustment process which takes a very long time to disappear due to the persistence observed through the fractionally differencing parameter d , (which is 0.7), and the autoregressive parameter (which is also high, 0.75). That suggests that unemployment is a nonstationary series with shocks taking a very long time to decay, and there is evidence that the main shocks which have affected it in the last 30 years are fluctuations in real oil prices and real interest rates.

Conclusions

In this article we have examined the underlying dynamics affecting the UK unemployment. However, instead of using the classical approaches based on $I(0)$ stationarity or $I(1)$ cointegrating relationships, we have gone throughout a new different approach based on fractionally integrated models. This is an important development since it allows for the possibility that unemployment is highly persistent. Hence, it allows us to test whether unemployment behaviour is due to extreme persistence to a limited set of shocks, rather than changes in its equilibrium.

Looking at the univariate behaviour of unemployment, we find strong evidence in favour of a unit root. Estimating d within a fractionally integrated ARMA (ARFIMA)

model, the null hypothesis of a unit root was almost never rejected and the ARFIMA(0, 1.18, 3) specification was chosen according to the likelihood criteria. Testing the order of integration of unemployment with the tests of Robinson (1994), the unit root hypothesis was also not rejected, though other alternatives, with d slightly smaller or higher than one were also plausible in some cases.

Including weakly exogenous regressors produced a different picture. The important determinants of unemployment seem to be the real oil prices lagged five periods and the current real interest rate, and in these cases the order of integration of unemployment was found to be smaller than 1 but higher than 0.50. That means that unemployment may be modelled as a nonstationary series with a strong component of mean-reverting behaviour, and this strongly suggests that shocks affecting it take a very long time to disappear.

The next step in this work is modelling prices and real interest rates stochastically and then, along with unemployment, in terms of the so-called fractional cointegration structure. This area, which is relatively new in econometrics, (a recent contribution here is Marinucci and Robinson, 1999), may lead to yet improved ways of explaining the adjustment process of unemployment due to variation in oil prices and real interest rates. Work in this direction is now in progress.

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