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Deterministic seasonality versus seasonal fractional integration

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ABSTRACT

We propose in this article the use of a testing procedure due to Robinson (1994) for testing deterministic seasonality versus seasonal fractional integration. A new statistic, based on the score principle, is developed to simultaneously test both the order of integration of the seasonal component and the need of seasonal dummies. Both tests have standard null and local limit distributions. However, finite-sample critical values of the tests are computed, and experiments based on Monte Carlo show that the sizes of the asymptotic tests are too large, these larger sizes being also associated with some superior rejection frequencies compared with the finite-sample-based tests. Using quarterly data for real consumption and income in Canada, the UK and Japan, the results show that both variables are seasonally fractionally integrated for the three countries without need of deterministic seasonal dummies. We also find evidence that the series may be seasonally fractionally cointegrated.

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1. INTRODUCTION

Modelling the seasonal component of macroeconomic time series has been a major focus of attention in recent years. Deterministic models based on seasonal dummy variables were initially adopted. Later on, it was observed that the seasonal component of many series changed over time and stochastic approaches based on seasonal differencing (see e.g. Box and Jenkins, 1970) were proposed. In recent years, seasonal-difference models have been extended to allow for other types of long memory behaviour, in particular, allowing seasonal fractional integration. For the purpose of the present paper, we assume that $\{u_t, t = 0, \pm 1, \ldots\}$ is an I(0) process, defined as a covariance stationary process with spectral density function, which is bounded and bounded away from zero at any frequency on the interval $[0, \pi]$.[1] We can consider the model:

$$(1 - L^s)^d x_t = u_t, \qquad t = 1, 2, \ldots \tag{1}$$

$$x_t = 0, \qquad t \leq 0, \tag{2}$$

where s is the number of time periods in a year, L^s is the seasonal lag operator ($L^s x_t = x_{t-s}$) and where d can be any real number. Note that the fractional polynomial can be expressed in terms of its Binomial expansion:

$$(1 - L^s)^d = \sum_{j=0}^{\infty} \binom{d}{j} (-1)^j L^{sj},$$

such that

$$(1 - L^s)^d x_t = x_t - d\, x_{t-s} + \frac{d(d-1)}{2} x_{t-2s} - \frac{d(d-1)(d-2)}{6} x_{t-3s} + \ldots.$$

for any real d. Clearly, if d = 0 in (1), $x_t = u_t$, and a weakly autocorrelated process is allowed for. However, for d > 0 in (1), x_t is said to be a seasonal long memory process, so-called because of the strong association (in the seasonal structure) between observations widely

[1] In other words, $0 < f(\lambda) < \infty$, where $f(\lambda)$ is the spectral density function of u_t.

separated in time.[2] The notion of fractional processes with seasonality was initially suggested by Abrahams and Dempster (1979) and Jonas (1981), and extended in a Bayesian framework by Carlin et al. (1985) and Carlin and Dempster (1989). Porter-Hudak (1990) applied a seasonally fractionally integrated model like (1) to quarterly US monetary aggregates and other recent empirical applications can be found, for example, in Silvapulle (1995), Ooms (1997) and Gil-Alana and Robinson (2001).

The outline of the paper is as follows: Section 2 describes a version of the tests of Robinson (1994) for testing the order of integration of the seasonal component in raw time series, with the possibility of including seasonal dummy variables in the original model. Section 3 presents a joint test statistic, based on Robinson (1994), for simultaneously testing the order of integration and the need of seasonal dummy variables. Finite-sample critical values of tests of Sections 2 and 3 are also computed in this section and Monte Carlo experiments are conducted to check the sizes and the power properties of the tests in finite samples. In Section 4 the tests are applied to the real consumption and income series of the UK, Canada and Japan while Section 5 contains some concluding comments and extensions.

2. THE TESTS OF ROBINSON (1994) AND SEASONALITY

Let's suppose we have quarterly data, (i.e., s = 4), and assume that $\{y_t, t = 1, 2, \ldots, T\}$ is the time series we observe. Let's consider the following model,

$$y_t \;=\; \beta_0 \;+\; \sum_{i=1}^{3} \beta_i \, D_{it} \;+\; x_t, \qquad t \;=\; 1, 2, \ldots \tag{3}$$

$$(1 \;-\; L^4)^d \, x_t \;=\; u_t, \qquad t \;=\; 1, 2, \ldots, \tag{4}$$

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[2] A seasonal long memory process is defined as a process with a singulatiry in the spectral density function at one or more seasonal frequencies on the interval (0, π].

where $\beta = (\beta_0, \beta_1, \beta_2, \beta_3)'$ is a (4x1) vector of unknown parameters; D_{1t}, D_{2t} and D_{3t} are the seasonal dummy variables, i.e., $D_{it} = 1I(t \in \text{quarter } i)$[3] and u_t is I(0). Based on (3) and (4), Robinson (1994) proposed a Lagrange Multiplier (LM) test of

$$H_o: \quad d = d_o \tag{5}$$

for any real value d_o. Specifically, the test statistic is given by

$$\hat{R} = \frac{T}{\hat{A}} \frac{\hat{a}^2}{\hat{\sigma}^4}, \tag{6}$$

where

$$\hat{a} = \frac{-2\pi}{T} \sum_j^* \psi(\lambda_j)\, g(\lambda_j;\hat{\tau})^{-1} I(\lambda_j); \qquad \hat{\sigma}^2 = \sigma^2(\hat{\tau}) = \frac{2\pi}{T} \sum_j^* g(\lambda_j;\hat{\tau})^{-1} I(\lambda_j),$$

$$\hat{A} = \frac{2}{T} \left(\sum_j^* \psi(\lambda_j)^2 - \sum_j^* \psi(\lambda_j)\hat{\varepsilon}(\lambda_j)' \times \left(\sum_j^* \hat{\varepsilon}(\lambda_j)\hat{\varepsilon}(\lambda_j)' \right)^{-1} \times \sum_j^* \hat{\varepsilon}(\lambda_j)\psi(\lambda_j) \right)$$

$$\psi(\lambda_j) = \log\left| 2\sin\frac{\lambda_j}{2} \right| + \log\left(2\cos\frac{\lambda_j}{2} \right) + \log\left| 2\cos\lambda_j \right|; \qquad \hat{\varepsilon}(\lambda_j) = \frac{\partial}{\partial \tau} \log g(\lambda_j;\hat{\tau});$$

$I(\lambda_j)$ is the periodogram of $\hat{u}_t$ defined as:

$$\hat{u}_t = (1-L^4)^{d_o} y_t - \hat{\beta}' w_t; \quad \hat{\beta} = \left(\sum_{t=1}^{T} w_t\, w_t' \right)^{-1} \sum_{t=1}^{T} w_t\, (1-L^4)^{d_o} y_t; \quad w_t = (1-L^4)^{d_o}(1, D_1, D_2, D_3)',$$

evaluated at $\lambda_j = 2\pi j/T$ and g is a known function coming from the spectral density function of $\hat{u}_t$: $f(\lambda;\sigma^2;\tau) = \frac{\sigma^2}{2\pi} g(\lambda;\tau)$, with $\hat{\tau} = \arg\min_{\tau \in T^*} \sigma^2(\tau)$, with T^* as a compact subset of the R^q Euclidean space. Note that these tests are purely parametric and therefore, they require specific modelling assumptions to be made regarding the short memory specification of u_t. Thus, if u_t is an AR process of form $\phi(L)u_t = \varepsilon_t$, $g = |\phi(e^{i\lambda})|^{-2}$, with $\sigma^2 =$

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[3] $I(x)$ is the indicator function: $I(x) = 1$ if $t \in x$, 0 otherwise.

$V(\varepsilon_t)$, so that the AR coefficients are function of τ. Also, if u_t is white noise, $g \equiv 1$ and $\hat{A}$ in (5) becomes:

$$\frac{2}{T}\sum_{j}^{*}\psi(\lambda_j)^2, \qquad (7)$$

which can be approximated, for large T, by $\pi^2/6 \approx 1.645$. Finally, the summation on * in the above expressions are over $\lambda \in M$ where $M = \{\lambda: -\pi < \lambda < \pi, \lambda \notin (\rho_l - \lambda_1, \rho_l + \lambda_1), l = 1, 2, \ldots, s\}$, such that ρ_l, $l = 1, 2, \ldots, s < \infty$ are the distinct poles of $\psi(\lambda)$ on $(-\pi, \pi]$.[4]

Based on (5), Robinson (1994) established that under certain regularity conditions,

$$\hat{R} \rightarrow_d \chi_1^2 \quad as \quad T \rightarrow \infty, \qquad (8)$$

and this holds independently of the type of I(0) disturbances used for u_t in (4). Thus, we are in a classical large-sample testing situation by reasons described in Robinson (1994). A 100α%-level test of (5) against H_a: $d \neq d_o$ will reject H_o (5) if $\hat{R} > \chi^2_{1,\alpha}$, where $\Pr ob(\chi_1^2 > \chi^2_{1,\alpha}) = \alpha$. Furthermore, he also showed that the test is efficient in the Pitman sense, i.e., that against local alternatives of form: H_a: $d = d_o + \delta T^{-1/2}$, for $\delta \neq 0$, $\hat{R}$ has a limit distribution given by a $\chi_1^2(\nu)$, with a non-centrality parameter, ν, which is optimal under Gaussianity of u_t. An empirical application of this version of Robinson's (1994) tests can be found in Gil-Alana and Robinson (2001).

Let's suppose now that we want to investigate if the seasonal component of a given time series is deterministic or alternatively, stochastically specified in terms of integrated processes. We can test H_o (5) with $d_o = 0$ in (3) and (4). Then, the model becomes exclusively (3) and the non-rejections of H_o (5) will imply, in this case, that the seasonal component is deterministic and thus, based exclusively on the seasonal dummy variables. On the other hand, testing H_o (5) for values of $d_o > 0$ and imposing $\beta_i = 0$ for i = 1,2 and 3 a priori in (3), the non-rejection values will indicate that the seasonal component is stochastic, either with unit roots (if $d_o = 1$) or with fractional ones (if $d_o \neq 1$). Furthermore, we can also test for seasonal fractional integration incorporating the seasonal dummies in (3), as well as including stationary autoregressions for the seasonal component. In the following section, we present a joint test statistic for testing simultaneously the need of the seasonal dummies and the order of integration of the seasonal component of the series.

3. A JOINT TEST OF SEASONALITY AND THE ORDER OF INTEGRATION

Gil-Alana and Robinson (1997) propose a joint test for testing the need of a linear time trend and the order of integration in a given time series at the zero frequency. In this section, a similar test is proposed but, instead of looking at the long run or zero frequency, we concentrate on the seasonal component of the series.

We can consider the model given by (3) and (4) and test the null hypothesis:

$$H_o : \ d = d_o \quad and \quad \beta_i = 0 \quad i = 1, 2, 3, \tag{9}$$

against the alternative:

$$H_o : \ d \neq d_o \quad or \quad \beta_i \neq 0 \quad for\ any \ \ i = 1, 2, 3. \tag{10}$$

To be slightly general, consider the regression model (3) with $\beta = (\beta_0, \beta_D')'$, where β_D is a (3×1) vector of coefficients corresponding to the seasonal dummies, and we want to test: H_o: $d = d_o$ and $\beta_D = \beta_{Do}$. Then, a Lagrange Multiplier (LM) statistic may be shown to be:

$$\tilde{S} = \tilde{R} + \sum_{t=1}^{T} \tilde{u}_t w_{2t}' \left(\sum_{t=1}^{T} w_{2t} w_{2t}' - \sum_{t=1}^{T} w_{2t} w_{1t} \times \left(\sum_{t=1}^{T} w_{1t}^2 \right)^{-1} \times \sum_{t=1}^{T} w_{1t} w_{2t}' \right)^{-1} \sum_{t=1}^{T} w_{2t} \tilde{u}_t \tag{11}$$

$w_{1t} = (1 - L^4)^{d_o} 1_T$, where 1_T is a (Tx1) vector of 1's and $w_{2t} = (1 - L^4)^{d_o} [D_{1t}, D_{2t}, D_{3t}]'$,

[4] Note that in the special case of d = 1, $w_{it} = 0$ for t > 4. However, $w_{0t} = (1 - L^4) 1_t = 1$ for t = 1, …, 4, and $w_{11} = w_{22} = w_{33} = 1$.

$$\tilde{u}_t = (1-L^4)^{d_o} y_t - \tilde{\beta}_0 w_{1t} - \beta_{Do}' w_{2t}; \qquad \tilde{\beta}_0 = \left(\sum_{t=1}^{T} w_{1t}^2\right)^{-1} \sum_{t=1}^{T} w_{1t} (1-L^4)^{d_o} y_t,$$

and $\tilde{R}$ as in (6) but using the $\tilde{u}_t$ just defined. (The derivation of the statistic is a simple exercise based on the score principle and using Appendix A in Robinson, 1994, and Gil-Alana and Robinson, 1997). Then, under H_o (9), $\tilde{S} \rightarrow_d \chi_4^2$ as $T \rightarrow \infty$, and we would compare (11) with the upper tail of the χ_4^2 distribution. However, we know that in finite samples, the empirical distribution of the tests of Robinson (1994) can vary substantially from the asymptotic results, (see e.g. Gil-Alana, 2000). Thus, we have computed, in Table 1, finite-sample critical values of both statistics, $\hat{R}$ in (6) and $\tilde{S}$ in (11).

(Table 1 about here)

In both cases we generate Gaussian series obtained by the routines GASDEV and RAN3 of Press, Flannery, Teukolsky and Vetterling (1986) with 50,000 replications each case, computing $\hat{R}$ in (6) and $\tilde{S}$ in (11) in a model given by (3) and (4). Due to the inclusion of the seasonal dummies in (3), the critical values will be affected by the order of integration in (4). Thus, we calculate the critical values for d = 0, 0.25, …, (0.25), …, 1.75 and 2, with sample sizes equal to 48, 96 and 120 and nominal sizes of 5% and 1%.

We see in Table 1 that for both statistics, the finite-sample critical values are much higher than those given by the χ^2 distributions, especially if the sample size is small. This implies that when testing the nulls (5) and (9) against the alternatives: H_a: $d \neq d_o$ and (10) with the asymptotic critical values, the tests will reject the null more often than with the finite-sample ones. We went deeper into the examination of these results and observed that the large numbers obtained for the finite-sample critical values were due to the fact that the quantity (7) required in (6) and (11) converges to its asymptotic value (1.645) very slowly. This quantity is 0.989 if T = 48; it is 1.216 if T = 96; it becomes 1.293 if T = 120, and only approximates 1.645 when T is higher than 300. (e.g., 1.624 if T = 360). Thus, the large numbers observed for the finite-sample values when T is small are due in large part to the small convergence of $\hat{A}$ to its asymptotic value.

We next examine the sizes and the power properties of the tests in finite samples, comparing the results using the finite-sample critical values obtained in Table 1 with those based on the asymptotic results. Tables 2 and 3 report the rejection frequencies of $\hat{R}$ and $\tilde{S}$ first, supposing that there is no need of seasonal dummies (i.e., imposing $\beta_1 = \beta_2 = \beta_3 = 0$ a priori, in Table 2), then including the dummy variables in the regression model (3), (i.e., with all coefficients unknown, in Table 3).

In Table 2, we assume that the true model is given by

$$y_t \;=\; 1 \;+\; x_t; \qquad (1 \;-\; L^4)\, x_t \;=\; \varepsilon_t,$$

with white noise ε_t and look at the rejection frequencies of $\hat{R}$ and $\tilde{S}$ in a model given by (3), (4), testing (5) and (9) with d_o = 0, 0.25, …, (0.25), …, 1.75 and 2 for a nominal size of 5% and the same sample sizes as in Table 1. Thus, the rejection frequencies corresponding to d_o = 1 will indicate the sizes of the tests since the true model contains seasonal unit roots and, in case of $\hat{R}$ in (6), the estimated β's should be around 0. We observe that for both test statistics the sizes of the asymptotic tests are too large in all cases, especially for the joint statistic $\tilde{S}$, though they improve slightly as we increase the number of observations. The higher sizes of the asymptotic tests compared with the finite-sample ones are also associated with some superior rejection frequencies, being higher the differences when we are close to the null d = 1. Looking at the results with T = 48, we see that the power of $\hat{R}$ is extremely low, especially when using the finite-sample critical values. This is not at all surprising noting that $\hat{R}$ assumes the inclusion of seasonal dummy variables which are not present in the true model. In that respect, the power of $\tilde{S}$ (which assumes no dummies under the null) is higher, though inferior with the finite-sample values than with the asymptotic results. Increasing the sample size (e.g. T = 120), the results of both tests for both types of critical values improve considerably, the rejection probabilities being competitive in all cases when the alternatives are far away from the null. It may finally be remarkable the fact that the power of the tests is not symmetric, especially for the case of $\hat{R}$. For example, when d_o is below 1, the rejection probabilities of $\hat{R}$ are much smaller than when d_o is above 1 by the same magnitude. This happens for all sample sizes and the same occurs with the joint test $\tilde{S}$, though the differences are here smaller.

(Tables 2 and 3 about here)

Table 3 assumes that the true model is given by:

$$y_t \;=\; 1 \;+\; D_{1t} \;+\; 2D_{2t} \;+\; 3D_{3t} \;+\; x_t; \quad (1 \;-\; L^4)x_t \;=\; \varepsilon_t, \tag{12}$$

and we perform the same experiment, i.e., computing $\hat{R}$ and $\tilde{S}$ for the same type of alternatives as in Table 2. Thus, the rejection frequencies of $\hat{R}$ with $d_o = 1$ will indicate the size of the test while the rejection probabilities of $\hat{R}$ for $d_o \neq 1$ and of $\tilde{S}$ for any d_o will give us information about the power of the tests. Surprisingly, the results for $\hat{R}$ are practically the same as in Table 2. That means that Robinson's (1994) tests have not much power in relation to the seasonal dummy variables, which makes the joint test statistic $\tilde{S}$ in (11) useful when describing these situations. Looking at the rejection frequencies of $\tilde{S}$, we see that they are very high when using the asymptotic critical values even if the sample size is small. Using the finite-sample ones, they are small for T = 48 if d_o is around 1, however, increasing T, they improve considerably, being higher than 0.900 if $d_o \leq 0.50$ or if $d_o \geq 1.50$ with T = 120. The same experiment was also conducted allowing different coefficients for the dummy variables in (12) and the same conclusions as those reported here were obtained.[5]

4. AN EMPIRICAL APPLICATION

We analyse in this section quarterly, seasonally unadjusted data,[6] corresponding to the consumption and income series for the UK, Canada and Japan. For the UK, the time period is 1955q1-1984q4; for Canada 1960q1-1994q4; and for Japan, 1961q1-1987q4. The data for Canada were retrieved from the CANSIM Statistics Canada database, and the UK and Japanese series were taken from Gil-Alana and Robinson (2001). Consumption is measured as the log of the total real consumption while income is in all cases the log of the total personal disposable income. The series for the UK and Japan were respectively analysed in Hylleberg, Engle, Granger and Yoo (HEGY, 1990) and in Hylleberg, Engle, Granger and Lee (HEGL, 1993) studying the seasonal integrated and cointegrated structure. The same series for the two countries were also examined in Gil-Alana and Robinson (2001), extending the analysis of HEGY (1990) and HEGL (1993) to the fractional case. The latter paper, however, does not consider deterministic dummy variables for describing the seasonal component and thus, this paper improves Gil-Alana and Robinson (2001) in that respect.

Denoting any of the series y_t, we employ throughout model (3) and (4) with white noise u_t, testing initially H_o (5) for values d_o = 0.00, 0.25, …, (0.25), …, 1.75 and 2.00. Table 4 reports values of the test statistic $\hat{R}$ in (6). We see that for the UK series, H_o (5) cannot be rejected if d_o ranges between 1 and 2, the lowest statistic being achieved in both

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[5] A similar experiment was conducted, testing the null $d_o = 1$ & $\beta_D = (1, 2, 3)'$ in model (12). Finite-sample critical values were obtained and they were very similar to those reported in the right hand side of Table 1, and the rejection frequencies were relatively high (with T = 120) even for alternatives of form d = 0.75 (0.754) and d = 1.25 (0.889).

[6] By "quarterly", we mean 4 observations per year, and "seasonally unadjusted data" means that the data have not been adjusted using seasonal statistical procedures.

series at $d_o = 1.50$. For the Canadian consumption and income, the only values of d_o where H_o cannot be rejected are $d_o = 1.75$ and 2, while for Japan, H_o (5) cannot rejected with $d_o = 1.75$ and 2 for consumption and with $d_o = 1.50$ and 1.75 for income. On the other hand, we also observe that H_o (5) always results in a rejection when $d_o = 0$, implying that at least for this simple case of white noise disturbances, the deterministic seasonal models are inappropriate for these series.

(Tables 4 and 5 about here)

In Table 5 we present the statistic $\tilde{S}$ in (11) for the same d_o values as before. We see a few more non-rejection cases than in Table 4 and in all them the non-rejection d_o's in that table form a proper subset of those in Table 5, suggesting that the seasonal dummies were unnecessary when modelling these series. The results in these two tables seem to indicate that a seasonal unit root is present in the UK consumption and income, while for Canada and Japan, higher orders of integration are observed. However, the significance of these results might be due in large part to unaccounted-for I(0) autocorrelated disturbances. Thus, Tables 6 and 7 report respectively the same statistics as in Tables 4 and 5 but allowing a seasonal autoregressive structure on u_t. We consider a seasonal AR(1) process of form:

$$u_t \;=\; \phi\, u_{t-4} \;+\; \varepsilon_t, \qquad (13)$$

with white noise ε_t, and though higher order seasonal and non-seasonal AR processes were also performed, the results were very similar to those reported in the tables. A problem here with the estimated AR's coefficients appears in that, though they entail roots that cannot exceed one in absolute value, they can be arbitrarily close to it, thus the disturbances being possibly non-significantly different from a seasonal unit root model. In order to solve this problem, we perform Dickey, Hasza and Fuller (DHF, 1984) tests on the residuals of the differenced regressions, and in those cases where the unit roots cannot be rejected, we do not report the statistics but mark with '—' in the tables. We see across Tables 6 and 7 that all of these cases occur when d_o is a very low number and close to 0. This is not at all surprising if we take into account that a model like (4) with d = 1 and white noise u_t is a very similar process, (though with very different statistical properties) to (13) with ϕ close to 1. The critical values for the AR(1) case with T = 120 were computed and though we do not report the values here, they were slightly higher than those given in Table 1. Starting with $\hat{R}$ in (6), we see in Table 6 that the non-rejection values occur at the same values of d_o for each series as in Table 4 with only two extra non-rejected d_o's corresponding to d_o = 1.50 for the Canadian income and d_o = 2 for the Japanese income.

(Tables 6 and 7 about here)

Similarly for the joint test, in Table 7, the non-rejections also coincide with those in Table 5 for the case of white noise disturbances, again with two extra non-rejected values, this time corresponding to d_o = 0.75 for the UK consumption and income series. In view of the results in these two tables, we have further evidence against the need of seasonal dummy variables for all the series in the three countries considered.[7]

As a final remark and following HEGY (1990), HEGL (1993) and Gil-Alana and Robinson (2001), we also investigate if consumption and income may be cointegrated. Using a very simplistic version of the "Permanent Income Hypothesis Theory" as discussed for example by Davidson, Hendry, Srba and Yeo (1978), we can consider a given cointegrating vector (1, -1) and look at the degree of integration of the difference between consumption and income. Thus, in Tables 8 and 9 we again perform $\hat{R}$ in (6) and $\tilde{S}$ in (11) this time on the differenced series, using both white noise and seasonal AR(1) disturbances. Starting with the case of white noise disturbances (in Table 8) we see that the non-rejection values of $\hat{R}$ take place when d_o ranges between 0.75 and 1.25. Thus, they are smaller by

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[7] In connnection with the short memory specification of u_t, the fact that the non-rejection values of d_o are the same whether u_t is white noise or AR may suggest that the former model for the disturbances is an adequate

about 0.50-0.75 than those given in Table 4. Similarly, using the joint statistic $\tilde{S}$, the non-rejection values of d_o also are smaller by approximately the same magnitude as before compared with Table 5 and, apart from the case of $d_o = 0.50$ where H_o (9) cannot be rejected now, all the remaining non-rejections occur at the same values of d_o as when using $\hat{R}$. Thus, we also find in this table evidence against deterministic seasonality as well as evidence of fractional cointegration at least for this case of white noise disturbances.

(Tables 8 and 9 about here)

Table 9 extends the results of Table 8 to the case of AR(1) disturbances. We see that the non-rejection values of d_o are smaller for both statistics, ranging between 0.50 and 1.50. Comparing the results here with those in Tables 6 and 7, we see that the orders of integration are again smaller for the differenced series, suggesting further evidence in favour of seasonal fractional cointegration.

5. CONCLUDING COMMENTS AND EXTENSIONS

A version of the tests of Robinson (1994) for testing the order of integration of the seasonal component in raw time series with the possibility of including seasonal dummy variables has been proposed in this article. Also, a joint test statistic for testing the order of integration and the presence of the dummy variables (i.e., $d = d_o$ & $\beta = 0$) was developed. Both tests have standard limit distributions under both the null and local alternatives. However, finite-sample critical values were computed and the values were much higher than those given by the χ^2 distributions. Monte Carlo experiments conducted across the paper showed that the tests based on the asymptotic results have much larger sizes than their corresponding nominal values, these larger sizes being also associated with some superior rejection frequencies compared with the finite-sample-based tests.

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way of specifying u_t. In fact, most of the AR coefficients in those cases where H_o (6) could not be rejected

The tests were applied to the consumption and income series of the UK, Canada and Japan. The results based on the tests of Robinson (1994) show that the orders of integration of the UK consumption and income widely fluctuate between 1 and 2 while the orders of integration of the Japanese and Canadian series are much higher than 1, in many cases being superior to 1.50. The joint test statistic was also performed on the series to see if the seasonal dummy variables were in fact required and the results showed that for all of them, the deterministic seasonals were inappropriate. Finally, we also performed the tests on the differenced series, $c_t - y_t$, to check if a seasonally fractionally cointegrated relationship might exist between consumption and income. The results here showed that the degree of integration of the differenced series was smaller than that of the original series, with the orders of integration fluctuating between 0.5 and 1.5, and thus supporting a very simplistic version of the Permanent Income Hypothesis. (Davidson et al., 1978).

The results obtained in this article are not directly comparable with those in Gil-Alana and Robinson (2001), the reason being that the latter paper does not include seasonal dummy variables in its regression model. In that respect, we found in this article certain evidence against the deterministic dummies and thus, the conclusions obtained in Gil-Alana and Robinson (2001) remains valid. HEGY (1990) and HEGY (1993) looked respectively at the UK and Japanese series exclusively in terms of seasonally integrated and cointegrated processes, and though they allow deterministic seasonality, they do not consider the possibility of seasonal fractional integration. Our results support the idea that the UK consumption and income may both be quarterly I(1) process (as in HEGY, 1990), however, unlike HEGL (1993) we found evidence against this hypothesis for the Japanese case. Finally, and similarly to all these authors, we also found support of the Permanent Income Hypothesis for the three countries considered.

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were insignificantly different from zero.

We should also mention that the test statistics presented in this article have nothing to do with the estimation of the fractional differencing parameter but simply generates computed diagnostics for different values of d. In this context, Ooms (1997) suggests Wald tests based on Robinson's (1994) model, using for the estimation a modified periodogram regression procedure of Hassler (1994), whose distribution is evaluated under simulation. Similar methods based on this and other procedures (e.g., Hosoya, 1997) can be applied to these and other macroeconomic time series.

The frequency domain set-up of the tests used in this article may result cumbersome for the practitioners. There also exist time domain versions of Robinson's (1994) tests (cf., Robinson, 1991, Silvapulle, 1995, Tanaka, 1999). However, the preference here for the frequency domain approach is motivated by the somewhat greater elegance that it affords especially if the disturbances are weakly autocorrelated.[8] The FORTRAN code used in this application is available from the author upon request.

This article can be extended in several directions. Clearly, we could have extended the tests to consider the case where the slope coefficients of some general regressors are 0 as well as the case where only a subset of the regression coefficients is tested. However, in the context of seasonality examined in this paper, we have considered more convenient to particularize the case of seasonal dummies to analyse the importance of these deterministic components. The seasonal differenced structure $(1 - L^4)^d$ can be decomposed (as in Gil-Alana and Robinson, 2001) into its long run component ($(1 - L)^d$) and the remaining seasonal structures ($(1 + L)^d$ and $(1 + L^2)^d$) and thus, we could test separately each of these components in the presence of seasonal dummies. For example, we can consider a model like (3) with

$$(1 - L)^d x_t = u_t, \qquad t = 1, 2, \ldots,$$

or alternatively

$$(1 + L)^d \, x_t = u_t, \qquad t = 1,2,...,$$

or

$$(1 + L^2)^d \, x_t = u_t, \qquad t = 1,2,...,$$

and test H_o (9) against (10) with the joint test described in Section 3. Note that the functional form of the test statistics will be the same as $\tilde{S}$ in (11) except for the differenced polynomial required to obtain w_{1t}, w_{wt} and the $\tilde{u}_t$. In addition, we can also test the significance of the dummies along with the orders of integration at each of the frequencies, i.e., testing H_o (9) in (3) and

$$(1 - L)^{d_1} (1 + L)^{d_2} (1 + L^2)^{d_3} \; x_t = u_t, \qquad t = 1, 2,...$$

with d = (d_1, d_2, d_3)' though here the limit distribution will be χ^2_6. Also, the seasonality can be extended to the monthly case, (see, Gil-Alana, 1999), and similarly to the quarterly structure, we can consider a model of form:

$$y_t = \beta_0 + \sum_{i=1}^{11} \beta_i D_{it} + x_t, \qquad t = 1,2,...,$$

$$(1 - L^{12})^d \, x_t = u_t, \qquad t = 1,2,...,$$

performing a similar version of Robinson's (1994) tests or alternatively the joint test of Section 3. Note that now the polynomial $(1 - L^{12})$ can be decomposed into

$$(1 - L)\,(1 + L + L^2 + ... + L^{11}) =$$

$$(1 - L)\,(1 + L)\,(1 + L^2)\,(1 + L + L^2)\,(1 - L + L^2)\,(1 + \sqrt{3}L + L^2)\,(1 - \sqrt{3}L + L^2),$$

implying respectively the presence of unit roots at

$$0;\; -1;\; \pm i;\; -\frac{1}{2}(1 \pm \sqrt{3}i);\; \frac{1}{2}(1 \pm \sqrt{3}i);\; -\frac{1}{2}(\sqrt{3} \pm i);\; \frac{1}{2}(\sqrt{3} \pm i)$$

---

[8] Seasonal long memory in the frequency domain has also been examined in Ooms and Hassler (1997) and Arteche (2002).

and then, more test statistics can de developed to test for the presence of unit and fractional roots at each of these frequencies. Cyclical roots as in Gil-Alana (2001) can also be considered in this context with deterministic components. Finally, in order to investigate if consumption and income are fractionally cointegrated, we could have applied Robinson's (1994) tests on the estimated residuals from the cointegrating regression instead of imposing a given vector as is the case in this paper. A problem here would appear in that the residuals would not actually be observed but obtained from the cointegrating regression, being possibly biased towards stationarity and thus, finite-sample critical values should be computed in this case. Work in all these directions is now under progress.

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TABLE 1					
Finite-sample critical values of $\hat{R}$ in (6) and $\tilde{S}$ in (11)					
Model: $y_t = \beta_0 + \sum_{i=1}^{3} \beta_i D_{it} + x_t; \; (1 - L^4)^d x_t = \varepsilon_t.$					
T = 48	d_o	$\hat{R}$ (H_o: d = d_o)		$\tilde{S}$ (H_o: d = d_o & $\beta_1 = \beta_2 = \beta_3 = 0$)	
		5%	1%	5%	1%
	0.00	6.82	9.31	13.91	19.03
	0.25	7.02	9.42	13.88	19.01
	0.50	7.40	9.96	13.81	18.69
	0.75	8.10	10.88	13.37	17.65
	1.00	8.40	11.43	13.10	17.33
	1.25	8.26	11.39	13.16	17.30
	1.50	8.15	11.44	13.14	17.41
	1.75	8.04	11.39	13.33	17.47
	2.00	7.94	11.18	13.42	17.69
T = 96	d_o	$\hat{R}$ (H_o: d = d_o)		$\tilde{S}$ (H_o: d = d_o & $\beta_1 = \beta_2 = \beta_3 = 0$)	
		5%	1%	5%	1%
	0.00	5.72	8.20	12.19	16.44
	0.25	5.84	8.37	12.23	16.58
	0.50	6.28	8.90	11.97	16.30
	0.75	6.68	9.38	11.76	15.76
	1.00	6.74	9.33	11.65	15.51
	1.25	6.57	9.13	11.77	15.46
	1.50	6.46	9.05	11.78	15.63
	1.75	6.37	8.96	11.79	15.67
	2.00	6.25	8.93	11.87	15.75
T = 120	d_o	$\hat{R}$ (H_o: d = d_o)		$\tilde{S}$ (H_o: d = d_o & $\beta_1 = \beta_2 = \beta_3 = 0$)	
		5%	1%	5%	1%
	0.00	5.37	8.05	11.59	16.35
	0.25	5.50	8.13	11.59	16.11
	0.50	5.95	8.60	11.54	15.53
	0.75	6.36	8.99	11.37	15.40
	1.00	6.23	8.95	11.30	15.35
	1.25	6.03	8.75	11.36	15.26
	1.50	5.92	8.66	11.38	15.42
	1.75	5.87	8.60	11.39	15.72
	2.00	5.77	8.64	11.36	15.71

The critical values of a χ_1^2 distribution are 3.84 at the 5% significance level and 6.63 at the 1% level. For the χ_4^2 distribution are 9.49 and 13.28 respectively.

TABLE 2					
Rejection frequencies of $\hat{R}$ and $\tilde{S}$ in (6) and (11)					
True model: $y_t = 1 + x_t; \quad (1-L^4)^{d_o} x_t = \varepsilon_t; \quad d_o = 1.$					
Alternative: $y_t = \beta_0 + \beta_1 D_{1t} + \beta_2 D_{2t} + \beta_3 D_{3t} + x_t; \ (1-L^4)^{d_o} x_t = \varepsilon_t.$					
	d_o	$\hat{R}$ (H_o: d = d_o)		$\tilde{S}$ (H_o: d = d_o & $\beta_1 = \beta_2 = \beta_3 = 0$)	
		FSCV	ASYMPTOTIC	FSCV	ASYMPTOTIC
T = 48	0.00	0.297	0.477	0.975	0.996
	0.25	0.089	0.246	0.868	0.967
	0.50	0.020	0.099	0.496	0.797
	0.75	0.044	0.234	0.122	0.514
	1.00	**0.052**	**0.365**	**0.050**	**0.486**
	1.25	0.092	0.534	0.103	0.665
	1.50	0.208	0.745	0.250	0.844
	1.75	0.380	0.879	0.439	0.944
	2.00	0.542	0.954	0.627	0.982
	d_o	$\hat{R}$ (H_o: d = d_o)		$\tilde{S}$ (H_o: d = d_o & $\beta_1 = \beta_2 = \beta_3 = 0$)	
		FSCV	ASYMPTOTIC	FSCV	ASYMPTOTIC
T = 96	0.00	0.959	0.973	0.999	1.000
	0.25	0.869	0.909	0.996	0.999
	0.50	0.367	0.510	0.784	0.941
	0.75	0.037	0.110	0.117	0.406
	1.00	**0.050**	**0.213**	**0.050**	**0.366**
	1.25	0.314	0.678	0.230	0.750
	1.50	0.771	0.957	0.599	0.968
	1.75	0.960	0.997	0.875	0.997
	2.00	0.995	1.000	0.969	0.999
	d_o	$\hat{R}$ (H_o: d = d_o)		$\tilde{S}$ (H_o: d = d_o & $\beta_1 = \beta_2 = \beta_3 = 0$)	
		FSCV	ASYMPTOTIC	FSCV	ASYMPTOTIC
T = 120	0.00	0.991	0.994	1.000	1.000
	0.25	0.961	0.974	0.999	0.999
	0.50	0.602	0.700	0.892	0.974
	0.75	0.068	0.137	0.138	0.432
	1.00	**0.050**	**0.175**	**0.050**	**0.332**
	1.25	0.451	0.741	0.304	0.796
	1.50	0.919	0.986	0.751	0.987
	1.75	0.996	0.999	0.961	1.000
	2.00	0.999	1.000	0.994	1.000

The nominal size is 5% in all cases. The sizes are in bold. FSCV means that we use the finite sample critical values obtained in Table 1.

TABLE 3					
Rejection frequencies of $\hat{R}$ and $\tilde{S}$ in (6) and (11)					
True model: $y_t = 1 + D_{1t} + 2D_{2t} + 3D_{3t} + x_t; \quad (1 - L^4)^{d_o} x_t = \varepsilon_t; \quad d_o = 1.$					
Alternative: $y_t = \beta_0 + \beta_1 D_{1t} + \beta_2 D_{2t} + \beta_3 D_{3t} + x_t; \; (1 - L^4)^{d_o} x_t = \varepsilon_t.$					
	d_o	$\hat{R}$ (H_o: d = d_o)		$\tilde{S}$ (H_o: d = d_o & $\beta_1 = \beta_2 = \beta_3 = 0$)	
		FSCV	ASYMPTOTIC	FSCV	ASYMPTOTIC
T = 48	0.00	0.298	0.478	0.987	0.997
	0.25	0.089	0.246	0.923	0.983
	0.50	0.024	0.098	0.701	0.911
	0.75	0.044	0.234	0.414	0.824
	1.00	**0.050**	**0.365**	0.342	0.838
	1.25	0.092	0.535	0.470	0.908
	1.50	0.201	0.746	0.659	0.962
	1.75	0.380	0.879	0.798	0.987
	2.00	0.541	0.954	0.886	0.996
	d_o	$\hat{R}$ (H_o: d = d_o)		$\tilde{S}$ (H_o: d = d_o & $\beta_1 = \beta_2 = \beta_3 = 0$)	
		FSCV	ASYMPTOTIC	FSCV	ASYMPTOTIC
T = 96	0.00	0.960	0.973	1.000	1.000
	0.25	0.867	0.912	0.996	0.999
	0.50	0.367	0.517	0.877	0.968
	0.75	0.037	0.112	0.421	0.770
	1.00	**0.050**	**0.213**	0.358	0.768
	1.25	0.314	0.678	0.622	0.935
	1.50	0.771	0.959	0.864	0.993
	1.75	0.961	0.997	0.968	0.999
	2.00	0.995	1.000	0.993	1.000
	d_o	$\hat{R}$ (H_o: d = d_o)		$\tilde{S}$ (H_o: d = d_o & $\beta_1 = \beta_2 = \beta_3 = 0$)	
		FSCV	ASYMPTOTIC	FSCV	ASYMPTOTIC
T = 120	0.00	0.992	0.995	1.000	1.000
	0.25	0.960	0.974	0.999	0.999
	0.50	0.601	0.700	0.935	0.986
	0.75	0.069	0.137	0.454	0.777
	1.00	**0.050**	**0.175**	0.364	0.749
	1.25	0.451	0.742	0.680	0.947
	1.50	0.920	0.987	0.926	0.997
	1.75	0.996	0.999	0.991	0.999
	2.00	0.999	1.000	0.999	1.000

The nominal size is 5% in all cases. The sizes are in bold. FSCV means that we use the finite sample critical values obtained in Table 1.

TABLE 4

Testing H_o (5) in (3) and (4) with $\hat{R}$ given by (6) with white noise disturbances

Series / d_o	0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
UK Consumption	158.32	119.71	74.85	24.69	**2.36’**	**0.10’**	**0.37’**	**1.93’**	**3.90’**
UK Income	152.12	113.33	67.58	18.18	**1.49’**	**0.03’**	**0.37’**	**1.65’**	**3.32’**
CAN Consumption	295.10	243.65	191.62	200.61	182.89	78.61	19.76	**0.24’**	**4.31’**
CAN Income	298.72	246.65	194.32	201.88	157.60	59.61	13.29	**0.15’**	**2.51’**
JAP Consumption	151.97	108.73	62.42	35.52	59.53	36.30	8.63	**0.16’**	**1.32’**
JAP Income	160.05	117.03	72.36	53.83	58.96	21.30	**1.00’**	**2.43’**	8.58

‘ and in bold: Non-rejection values at the 95% significance level. CAN and JAP mean respectively Canadian and Japanese series.

TABLE 5

Testing H_o (9) in (3) and (4) with $\tilde{S}$ given by (11) with white noise disturbances

Series / d_o	0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
UK Consumption	170.49	130.76	73.44	14.68	**0.98’**	**0.01’**	**0.78’**	**2.64’**	**4.72’**
UK Income	188.09	139.06	76.45	14.98	**1.60’**	**0.10’**	**0.37’**	**1.88’**	**3.80’**
CAN Consumption	292.60	242.38	190.38	193.84	154.38	53.28	**8.98’**	**0.36’**	**7.67’**
CAN Income	296.27	245.41	193.02	193.91	128.94	39.81	**5.19’**	**0.69’**	**6.70’**
JAP Consumption	138.95	103.42	59.70	30.42	32.64	14.10	**1.30’**	**0.73’**	**3.83’**
JAP Income	109.45	93.92	59.50	32.94	18.24	**3.04’**	**0.53’**	**4.34’**	**8.00’**

‘ and in bold: Non-rejection values at the 95% significance level. CAN and JAP mean respectively Canadian and Japanese series.

TABLE 6									
Testing H_o (5) in (3) and (4) with $\hat{R}$ given by (6) and seasonal AR(1) disturbances									
Series / d_o	0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
UK Consumption	--	--	13.86	12.35	**1.61’**	**0.76’**	**0.24’**	**0.001’**	**0.15’**
UK Income	--	--	13.49	10.39	**1.70’**	**0.79’**	**0.27’**	**0.01’**	**0.06’**
CAN Consumption	--	--	--	252.13	230.04	54.08	9.41	**4.08’**	**0.23’**
CAN Income	--	--	--	214.48	176.32	33.63	**7.08’**	**2.20’**	**0.01’**
JAP Consumption	--	--	71.22	66.72	55.08	24.34	12.60	**6.37’**	**1.29’**
JAP Income	--	--	--	50.31	45.54	14.19	**7.25’**	**1.18’**	**0.56’**

‘ and in bold: Non-rejection values at the 95% significance level. ‘—‘ means that the disturbances contain a seasonal unit root. CAN and JAP mean respectively Canadian and Japanese series.

TABLE 7									
Testing H_o (9) in (3) and (4) with $\widetilde{S}$ given by (11) and seasonal AR(1) disturbances									
Series / d_o	0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
UK Consumption	--	--	12.25	**9.18’**	**0.89’**	**0.42’**	**0.09’**	**0.04’**	**0.36’**
UK Income	--	--	12.90	**9.25’**	**1.32’**	**0.60’**	**0.18’**	**0.008’**	**0.11’**
CAN Consumption	--	--	--	201.39	178.07	34.30	**6.55’**	**1.94’**	**0.14’**
CAN Income	--	--	--	200.22	134.75	24.11	**7.22’**	**2.14’**	**0.07’**
JAP Consumption	--	--	50.76	42.29	28.50	12.95	**7.07’**	**2.09’**	**0.09’**
JAP Income	--	--	--	21.44	13.62	**6.77’**	**2.84’**	**0.26’**	**1.29’**

‘ and in bold: Non-rejection values at the 95% significance level. ‘—‘ means that the disturbances contain a seasonal unit root. CAN and JAP mean respectively Canadian and Japanese series.

TABLE 8

Testing H_o (5) in (3) and (4) with $\hat{R}$ given by (6) with white noise disturbances

Series / d_o	0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
UK: C_t - Y_t	134.91	63.08	9.37	**0.02’**	**1.75’**	**4.41’**	7.21	9.78	12.04
CANADA: C_t - Y_t	111.10	61.85	12.65	**0.01’**	**3.76’**	10.82	16.61	20.56	23.32
JAPAN: C_t - Y_t	108.06	69.22	23.84	**2.92’**	**0.32’**	**4.13’**	8.22	11.15	13.16

Testing H_o (9) in (3) and (4) with $\widetilde{S}$ given by (11) with white noise disturbances

Series / d_o	0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
UK: C_t - Y_t	53.72	38.10	**5.93’**	**0.19’**	**2.39’**	**5.21’**	**8.00’**	12.45	12.56
CANADA: C_t - Y_t	21.74	19.05	**7.69’**	**0.26’**	**8.05’**	17.90	25.19	30.16	33.63
JAPAN: C_t - Y_t	25.93	12.86	**2.26’**	**1.97’**	**0.09’**	**2.57’**	**5.87’**	18.50	20.41

‘ and in bold: Non-rejection values at the 95% significance level. CAN and JAP mean respectively Canadian and Japanese series.

TABLE 9

Testing H_o (5) in (3) and (4) with $\hat{R}$ given by (6) with AR(1) disturbances

Series / d_o	0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
UK: C_t - Y_t	--	--	**1.14’**	**0.10’**	**0.82’**	**1.63’**	**2.55’**	7.47	9.40
CANADA: C_t - Y_t	--	--	**0.11’**	**0.36’**	**2.42’**	**5.22’**	8.64	11.95	14.77
JAPAN: C_t - Y_t	--	--	**0.56’**	**1.46’**	**0.77’**	**0.04’**	**1.56’**	7.95	9.09

Testing H_o (9) in (3) and (4) with $\widetilde{S}$ given by (11) with AR(1) disturbances

Series / d_o	0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
UK: C_t - Y_t	--	10.01	**0.15’**	**0.42’**	**1.30’**	**2.16’**	**8.09’**	13.96	16.81
CANADA: C_t - Y_t	--	12.28	**6.77’**	**3.58’**	**3.91’**	**6.74’**	**10.58’**	14.46	18.08
JAPAN: C_t - Y_t	--	12.95	**1.92’**	**0.11’**	**0.46’**	**0.04’**	**8.80’**	12.35	14.00

‘ and in bold: Non-rejection values at the 95% significance level. ‘—‘ means that the disturbances contain a seasonal unit root. CAN and JAP mean respectively Canadian and Japanese series.