



Universidad  
de Navarra

Facultad de Ciencias Económicas y Empresariales

## **Working Paper nº 09/09**

Warming break trends and fractional integration in  
the northern, southern and global temperature  
anomaly series

Luis A. Gil-Alana  
Facultad de Ciencias Económicas y Empresariales  
Universidad de Navarra

Warming break trends and fractional integration in the northern, southern and global temperature anomaly series

Luis A. Gil-Alana

Working Paper No.09/09

October 2009

### ABSTRACT

This paper deals with the estimation of time trends in temperature anomaly series. However, instead of imposing that the estimated residuals from the time trends are  $I(0)$  stationary, we allow them to be fractionally integrated. In this context, a new procedure for testing fractional integration with segmented trends is applied to the northern, southern and global temperature anomaly series. The results show that the three series are fractinally integrated, and the warming effects are substantially higher after the break in all cases.

Luis A. Gil-Alana  
Universidad de Navarra  
Depto. Economía  
Campus Universitario  
31080 Pamplona  
[alana@unav.es](mailto:alana@unav.es)

## 1. Introduction

The main purpose of this article is to examine if there are significant trends in the global and hemispheric temperature anomaly series. Denoting a time series for temperature anomalies by  $y_t$ , the standard approach is to employ a simple linear regression model of form:

$$y_t = \alpha + \beta t + u_t, \quad t = 1, 2, \dots, \quad (1)$$

testing the significance of the estimated slope coefficient for  $\beta$  in (1). It is not uncommon to find estimates of  $\beta$  based on Ordinary Least Squares (OLS). However, statistical inference based on standard  $t$  or  $F$  statistics is only valid here if (1) satisfies some restrictive conditions, which are rarely satisfied in temperature time series. Thus, for example, the error term in (1) should be uncorrelated. If there is some autocorrelation in (1), the autoregressive AR(1) process

$$u_t = \rho u_{t-1} + v_t, \quad t = 1, 2, \dots, \quad (2)$$

has been widely employed in the climatological community because of its relation with the stochastic first order differential equation. In this case, Generalized Least Squares (GLS) is usually adopted for the estimation of  $\beta$ .<sup>1</sup> Nevertheless, in both cases (i.e., no autocorrelation and AR(1)  $u_t$ ) the series is supposed to be stationary  $I(0)$  once the time trend has been removed. If the detrended series is not  $I(0)$  the classic alternative is to assume that the series is nonstationarity or that it contains a unit root, also called integration of order 1 ( $I(1)$ ), and  $u_t$  in (1) is then described as:

$$u_t = u_{t-1} + v_t, \quad t = 1, 2, \dots,$$

where  $v_t$  is supposed to be  $I(0)$ . Thus, it is crucial to determine if the error term in (1) is stationary  $I(0)$  or nonstationary  $I(1)$ .<sup>2,3</sup>

The most common used indicator of climate change is the surface air temperature and there is a vast amount of papers examining the trends in global and regional mean temperatures over time (Ghil and Vautard, 1991; Hasselmann, 1993; Schlesinger and Ramankutty, 1994; North and Kim, 1995; North et al., 1995) and in global patterns of temperature change (Santer et al., 1995; Hegerl et al., 1996, 1997; Jones and Hegerl, 1998, etc.). Most of these papers conclude that global mean annual surface temperatures have increased between 0.3°C and 0.6°C during the last 150 years (Hansen and Lebedeff, 1987; Nicholls et al. 1996; Jones et al. 1997). In most of these articles, it is assumed  $I(0)$  stationarity for  $u_t$  in (1), while unit roots are found in global surface temperature in Stern and Kaufmann (2000) and Kaufmann et al. (2005).

In this article we extend the previous literature in two aspects. First, we consider fractional degrees of integration for the disturbance term  $u_t$  in (1). This is clearly more general than the standard approaches based on  $I(0)/I(1)$  specifications, also including these two approaches as particular cases of interest. Moreover, we permit the existence of a structural break, which is endogenously determined by the model, and this allows us to consider the case of segmented trends in the temperature series. These two issues are relevant in the analysis of the warming effect. First, to correctly determine the degree of integration of the error term is crucial for the estimation of the time trend coefficients: misspecification of the degree of dependence leads to inconsistent estimates of these coefficients. On the other hand, it is plausible to assume that the degree of persistence of the series has changed across the years, leading also to different estimates of the time trend coefficients across subsamples.

Fractionally integrated models have been employed to analyze temperature time series in previous research. Thus, for example, Gil-Alana (2005) examines the

temperatures in the northern hemisphere from a fractional viewpoint. He uses data from 1854 to 1999. The present work goes beyond that paper in various aspects: first, it looks not only at the northern temperatures but also at global and southern hemisphere temperature anomalies, for a time period that goes from 1850 through 2006. Moreover, including a structural break, it allows us to consider the possibility of time-varying trend coefficients along with different degrees of persistence for each subsample. The remaining of the paper is structured as follows: Section 2 presents the model considered in the paper. Section 3 describes the time series data; Section 4 is devoted to the empirical work on the temperature anomaly series, while Section 5 concludes the paper.

## 2. Fractional integration and segmented trends

For the purpose of the present work we define an  $I(0)$  process as a covariance stationary process with spectral density that is positive and finite at the zero frequency. Thus, it may be a simple white noise process, but it may also include some type of weak dependence (e.g. AR) structure. When first differences are required to achieve  $I(0)$ , the series is said to be  $I(1)$ . However, the  $I(0)$  and the  $I(1)$  models are merely two particular cases of a much more general class of processes called fractionally integrated or  $I(d)$  processes, where  $d$ , (the number of differences required to get  $I(0)$ ), may be a real value. In other words, we say that  $u_t$  is  $I(d)$  if:

$$(1 - L)^d u_t = v_t, \quad t = 1, 2, \dots, \quad (3)$$

with  $u_t = 0, t \leq 0$ , where  $L$  is the lag-operator (i.e.,  $Lu_t = u_{t-1}$ ), and  $v_t$  is  $I(0)$ .<sup>4</sup> Note that the polynomial in the left-hand-side in (3) can be expressed in terms of its Binomial expansion, such that, for all real  $d$ ,

$$(1-L)^d = \sum_{j=0}^{\infty} \psi_j L^j = \sum_{j=0}^{\infty} \binom{d}{j} (-1)^j L^j = 1 - d L + \frac{d(d-1)}{2} L^2 - \dots \quad (4)$$

and (3) can be written as:

$$u_t = d u_{t-1} - \frac{d(d-1)}{2} u_{t-2} + \dots + v_t.$$

If  $d$  is an integer value,  $u_t$  will be a function of a finite number of past observations, while if  $d$  is real,  $u_t$  depends strongly upon values of the time series far away in the past. (See, e.g., Granger and Ding, 1996; Dueker and Asea, 1998). Moreover, higher the  $d$  is, the higher will be the level of association between the observations. Examples of this type of model in meteorological time series data are among others the papers of Bloomfield (1992); Smith (1993); Lewis and Ray (1997); Pethkar and Selvam (1997); Koscielny-Bunde et al. (1998); Pelletier and Turcotte (1999); Percival et al. (2004); Maraun et al. (2004) and Gil-Alana (2003, 2005).

In the first part of the empirical application carried out in the following section, we consider models of form as in (1) – (3), testing the significance of the slope coefficient in a model where the disturbance term may be fractionally integrated. In the second part, we extend the model to allow for a segmented trend. In particular, we consider models of form:

$$y_t = \alpha_1 + \beta_1 t + x_t; \quad (1-L)^{d_1} x_t = u_t, \quad t = 1, \dots, T_b \quad (5)$$

$$y_t = \alpha_2 + \beta_2 t + x_t; \quad (1-L)^{d_2} x_t = u_t, \quad t = T_b + 1, \dots, T, \quad (6)$$

where the  $\alpha$ 's and the  $\beta$ 's are the coefficients corresponding respectively to the intercepts and the linear trends;  $d_1$  and  $d_2$  may be real values, and they are the orders of integration for each subsample,  $u_t$  is  $I(0)$  and  $T_b$  is the time of the break that is supposed to be unknown. Note that the model in equations (5) and (6) can also be written as:

$$(1 - L)^{d_1} y_t = \alpha_1 \tilde{l}_t(d_1) + \beta_1 \tilde{t}_t(d_1) + u_t, \quad t = 1, \dots, T_b, \quad (7)$$

$$(1 - L)^{d_2} y_t = \alpha_2 \tilde{l}_t(d_2) + \beta_2 \tilde{t}_t(d_2) + u_t, \quad t = T_b + 1, \dots, T, \quad (8)$$

where  $\tilde{l}_t(d_i) = (1 - L)^{d_i} 1$ , and  $\tilde{t}_t(d_i) = (1 - L)^{d_i} t$ ,  $i = 1, 2$ .<sup>5</sup>

The method presented here is based on the least squares principle. First we choose a grid for the values of the fractionally differencing parameters  $d_1$  and  $d_2$ , for example,  $d_{i0} = 0, 0.001, 0.002, \dots, 1$ ,  $i = 1, 2$ . Then, for a given partition  $\{T_b\}$  and given  $d_1, d_2$ -values,  $(d_{1o}, d_{2o})$ , we estimate the  $\alpha$ 's and the  $\beta$ 's by minimizing the sum of squared residuals,

$$\min \sum_{t=1}^{T_b} \left[ (1-L)^{d_{1o}} y_t - \alpha_1 \tilde{l}_t(d_{1o}) - \beta_1 \tilde{t}_t(d_{1o}) \right]^2 + \sum_{t=T_b+1}^T \left[ (1-L)^{d_{2o}} y_t - \alpha_2 \tilde{l}_t(d_{2o}) - \beta_2 \tilde{t}_t(d_{2o}) \right]^2$$

*w.r.t.*  $\{\alpha_1, \alpha_2, \beta_1, \beta_2\}$

in case of uncorrelated disturbances or using other methods like Generalized Least Squares (GLS) with autocorrelated  $u_t$ .

Let  $\hat{\alpha}(T_b; d_{1o}^{(1)}, d_{2o}^{(1)})$  and  $\hat{\beta}(T_b; d_{1o}^{(1)}, d_{2o}^{(1)})$  denote the resulting estimates for partition  $\{T_b\}$  and initial values  $d_{1o}^{(1)}$  and  $d_{2o}^{(1)}$ . Substituting these estimated values on the objective function, we have  $RSS(T_b; d_{1o}^{(1)}, d_{2o}^{(1)})$ , and minimizing this expression across all values of  $d_{1o}$  and  $d_{2o}$  in the grid we obtain:  $RSS(T_b) = \arg \min_{\{i,j\}} RSS(T_b; d_{1o}^{(i)}, d_{2o}^{(j)})$ . Then, the estimated break date,  $\hat{T}_k$ , is such that:  $\hat{T}_k = \arg \min_{i=1, \dots, m} RSS(T_i)$ , where the minimization is taken over all partitions  $T_1, T_2, \dots, T_m$ , such that  $T_i - T_{i-1} \geq |\epsilon T|$ . Then, the regression parameter estimates are the associated least-squares estimates of the estimated  $k$ -partition, i.e.,  $\hat{\alpha}_i = \hat{\alpha}_i(\{\hat{T}_k\})$ ,

$\hat{\beta}_i = \hat{\beta}_i(\{\hat{T}_k\})$ , and their corresponding differencing parameters,  $\hat{d}_i = \hat{d}_i(\{\hat{T}_k\})$ , for  $i = 1$  and  $2$ . Several Monte Carlo results based on the model in (5) and (6) are provided in Gil-Alana (2007). In that paper the author shows that the method performs relatively well even with small samples.

The possibility of segmented trends in temperature time series has not been widely examined so far. In a recent paper, however, Wu and Zhao (2007) develop a procedure for breaks in trends in means nonstationary models, and apply it to the global monthly temperatures from 1856 to 2000. They conclude there are no jumps in linear trends and suggest that a quadratic trend could be more appropriate for this dataset. This result is also in line with Rust (2003), who fitted a quadratic trend for the same dataset. Nevertheless, these authors do not consider the possibility of fractional integration, and it is now a well-known stylized fact that fractional integration and structural breaks are issues which are intimately related. Moreover, it seems reasonable for historical data as those analysed in this paper to assume that the degree of persistence in the data (and thus, the estimation of the time trend coefficients) have changed across the years.<sup>6</sup>

### **3. The temperature anomaly series**

The time series data analysed in this paper correspond to the global and hemispheric temperature anomaly series, obtained from the Climatic Research Unit (CRU) at the University of East Anglia, United Kingdom, and are available on line at the web-site: <http://www.ru.uea.ac.uk/data/temperature>. They are annual data from 1850 to 2006, and the anomalies are with respect to 1961-1990. These data are continually updated and expanded by P. Jones of the CRU with help from colleagues at the CRU and other institutions. Some of the earliest work in producing these series back to Jones et al.



(1986a,b,c), Jones (1988, 1994) and Jones and Briffa (1992). The data consist of surface air temperature data (land surface temperature) from over 3000 station records that have been corrected for non-climatic errors, such as station shifts and/or instrument changes (Jones, 1994). Coverage is denser over the more populated parts of the world, particularly United States, southern Canada, Europe and Japan, and is sparsest over the interior of the South American and African continents and over the Antarctic. The number of available stations was small during the 1850s, but increases to over 3000 stations during the 1951-1990 period. A review of the methods employed and data available can be found in Jones, Parker, Osborn and Briffa (2006).

Figure 1 displays the plots of the three series. The northern and southern temperature series show some general similarities, e.g., little sign of trends before 1900, a peak in the early 1940s, and the highest temperatures occurring after 1980. However, according to Jones et al. (2006): *“there are several notable differences between the two series: a steady period of warming is seen for the northern hemisphere from about 1910 through the mid-1940s. For the southern hemisphere there is less warming observed from 1910 through 1930, with sudden and rapid warming from 1930 through mid-1940s. The northern hemisphere records show gradual cooling from the mid-1940s through the mid-1970s, followed by rather steady temperature increases thereafter. The southern hemisphere shows an abrupt shift to cooler temperatures after 1945, quite variable temperatures until the mid-1960s, followed by a gradual increase over the remainder of the record.”*

**[Insert Figure 1 about here]**

For the global temperatures it is observed stable temperatures from the beginning of the record through about 1910, with relatively rapid and steady warming through the early 1940s, followed by another period of relatively stable temperatures through the mid-1970s. From this point onward, another rapid rise was observed. 1998 was the warmest year to date, followed by 2005, and nine of the ten warmest years in the sample occurred in the last ten years (1995-2006).

Jones et al. (2006) estimated linear trends on these three series based on simple linear regression techniques, and the results showed a warming trend of about  $0.69^{\circ}\text{C}$  for the global and northern hemisphere temperatures, and about  $0.70^{\circ}\text{C}$  for the southern temperatures anomaly series. Other studies using these and other variables for shorter time periods found values for the warming effect slightly smaller than those reported in Jones et al. (2006). Thus, for example, Gil-Alana (2005) found an estimate of about  $0.48^{\circ}\text{C}$  for the northern hemisphere anomalies for the time period 1854-2001. Smith (1993) applied a long memory model to warming data over a variety of sites in central England and the continental United States, and estimated the temperature increases to be between  $0.27$  and  $0.35^{\circ}\text{C}$ . Standard errors of these estimators ranged between  $0.19$  and  $0.31$ , however, suggesting a marginal, but not strongly significant increase in long-range temperatures. Using global temperature averages collected by Hansen and Lebedeff (1987), Smith and Chen (1996) proposed a joint estimation of the long memory parameter and the time trend, and found an estimate for the time trend of about  $0.55^{\circ}\text{C}$ .

#### **4. Empirical results on the temperature anomaly series**

First we suppose that there are no breaks in the data and thus, we consider a model of form as in (1) and (3), testing the null hypothesis

$$H_0: d = d_0, \quad (9)$$

for any real value  $d_0$ . Thus, under  $H_0$  (9) the model becomes:

$$y_t = \alpha + \beta t + u_t, \quad t = 1, 2, \dots, \quad (10)$$

$$(1 - L)^{d_0} u_t = v_t, \quad t = 1, 2, \dots, \quad (11)$$

and we test  $H_0$  (9) for  $d_0$ -values from 0 to 1 with 0.001 increments, using Robinson's (1994) parametric approach. Robinson's (1994) method does not require preliminary differencing; it allows us to test any real value  $d$  encompassing stationary and nonstationary hypotheses. Since this method is parametric, we need to specify a functional form for the disturbance term  $v_t$ , and first we suppose  $v_t$  is white noise. Then, an AR(1) structure is considered. In both cases we first suppose that  $\alpha = \beta = 0$  (i.e., we do not consider deterministic terms in the undifferenced regression (10)); then, an intercept is included (i.e.,  $\alpha$  unknown and  $\beta = 0$  a priori), and finally, we include an intercept and a linear time trend (i.e.,  $\alpha$  and  $\beta$  unknown).

**[Insert Table 1 about here]**

Table 1 shows the test results; the numbers in bold are the maximum likelihood estimates of  $d$  obtained with the Whittle function. Table 1 also shows the 95% confidence bands for the non-rejections of  $d_0$  using Robinson's (1994) parametric approach. The first noticeable feature observed in this table is that the  $I(0)$  hypothesis (i.e.,  $d = 0$ ) and the unit root model (i.e.,  $d = 1$ ) are both decisively rejected across all cases. Thus, we find strong evidence of fractional integration for the three series. Starting with the case of a white noise model for the disturbance term, we observe that the

estimates of  $d$  lie between 0.367 and 0.449 for the northern hemisphere; they are between 0.418 and 0.480 for the southern temperatures, and range between 0.452 and 0.516 for the global temperatures. Thus, the values lie in all cases close to the borderline between stationary and nonstationary processes ( $d = 0.5$ ). Also, if a linear trend is included, the orders of integration are substantially smaller for the three series considered. If we permit  $I(0)$  autocorrelation for the error term, the values are generally higher. The estimates are above 0.5 (i.e., in the nonstationary region) for the northern and global temperatures, while they are below 0.5 (and thus stationary) for the southern hemisphere.

**[Insert Table 2 about here]**

Table 2 displays for each series and each model the estimates of the coefficients associated to the time trend, along with the estimated value of  $d$ , and the AR coefficient. We observe that the coefficients for the time trend are all statistically significant. Thus, the warming trend coefficients are  $0.60^{\circ}\text{C}$  and  $0.54^{\circ}\text{C}$  for the northern hemisphere respectively for the cases of white noise and  $\text{AR}(1)$  disturbances; they are  $0.38^{\circ}\text{C}$  for the two cases in the southern hemisphere, and they are  $0.47^{\circ}\text{C}$  and  $0.44^{\circ}\text{C}$  for the global temperatures. These values are smaller than those reported in Jones et al. (2006) where the disturbance term was supposed to be  $I(0)$ . They are also slightly smaller than those given by Fomby and Vogelsang (2002). These authors proposed a test that is valid under wide distributional assumptions for the error term, including the two standard cases of  $I(0)$  and  $I(1)$  disturbances. According to these authors, temperatures have increased about  $0.5^{\circ}\text{C}/100$  yr., though they remark that if the analysis is restricted to the twentieth century data, many of the point trend-estimates are closer to  $0.6^{\circ}\text{C}$ .

**[Insert Figure 2 about here]**

Figure 2 displays the estimated trends for the three series. It is observed that the largest trend corresponds to the northern hemisphere temperatures, and for the three series the results are fairly similar independently of the way of modelling the disturbance term.

The results presented so far imply that the time trend has remained fixed across the sample period. This assumption may be unrealistic, especially taking into account that the balance between the greenhouse gases and the sulphur emissions has not remained stable across the years, and this has clearly an effect on the discontinuous changes observed in temperature time series. Thus, in what follows we consider the possibility of segmented trends and include one structural break at an unknown point in time, using for the estimation the procedure described in Section 2. The results, for the two cases of white noise and AR(1) disturbances, are displayed in Table 3.

**[Insert Table 3 about here]**

Starting with the white noise specification we observe that the break date takes place at 1964 for the northern hemisphere, the two time trends are statistically significant, and the orders of integration are 0.348 and 0.479 for the first and second subsamples respectively. For the southern and global temperatures, the breaks occur in the earlier part of the sample (1891 for the southern hemisphere and 1884 for the global

temperatures) and the time trends are not significant for the first subsamples. In both series the orders of integration are smaller in the second sub-samples.

If we permit an AR(1) process for the disturbance term, the results are displayed in the lower part of Table 3. For the hemispheric temperatures the breaks take place at approximately the same dates as in the white noise case, however, for the global temperatures, the break occurs now at 1973. The time trend coefficients are significant in practically all cases, the only exception being the southern temperatures during the first subsample, and the estimated values of  $d$  are smaller than 0.5 and thus stationary in all cases. In fact, for two cases (southern temperatures in the first subsample, and global temperatures during the second subsample) the estimated value of  $d$  is precisely 0, implying lack of long memory behaviour in these two cases.<sup>7</sup>

**[Insert Figures 3 and 4 about here]**

Figures 3 and 4 reproduce the segmented trends for the three series using white noise and AR(1) errors. Starting with the white noise case, (in Figure 3 and Table 4), we observe that for the northern temperatures, the warming effect changes from 0.54°C before 1964 to 1.75°C after that date; for the southern and global temperatures the warming effect is insignificant during the first subsamples, and is about 0.81°C (after 1891) for the southern temperatures, and about 0.75°C (after 1884) for the global temperatures.

Assuming that the disturbances follow an AR(1) process, (Figure 4 and Table 4), the change in the warming effect is even more significant: for the northern temperatures, it changes from 0.37°C (before 1963) to 2.84°C after the break; it is 0.74°C from 1891 for

the southern hemisphere, and the values are 0.19°C (before 1973) and 2.61°C (after 1973) for the global temperatures.

## 5. Conclusions

In this article we have examined the existence of significant trends in the global and hemispheric temperature anomaly series, annually, for the time period 1850-2006. However, instead of imposing a priori that the residuals from the estimated trends are stationary  $I(0)$ , we test this hypothesis by using fractional integration. The results strongly support the view that the northern, southern, and global temperatures are  $I(d)$  with  $d$  constrained between 0 and 1. Moreover, the existence of a structural break at an unknown point in time is also taken into account, examining the implications that this has on the orders of integration and the coefficients associated to the time trends.

**[Insert Table 4 about here]**

Table 4 summarizes the results in terms of the warming effects depending on the assumption made on the disturbance term. As expected, the results substantially differ from one model to another. Generally, the values are smaller if fractional orders of integration are taken into account. This is consistent with other empirical works that assume  $I(0)$  specifications for the error term, also suggesting that the results based on stationary  $I(0)$  are upward biased since this hypothesis is strongly rejected when fractional values are taken into account. If segmented trends are permitted, the values are higher during the second subsamples implying stronger warming effects during the latter parts of the samples.

The results presented in this paper indicate that the temperature anomaly time series are fractionally integrated, implying a strong degree of association between the observations widely separated in time. Nevertheless, the fact that in all cases the fractional differencing parameters are found to be strictly smaller than one also implies that the series are mean-reverting. Thus, in the event of an exogenous shock caused, for example, by exogenous forces, there is no strong need of policy actions since the series will return to their original trends sometime in the future. Another implication of these results is that the underlying trends have substantially increased in recent years, and which are the causes of such increase is an issue that will be examined in future papers.

This article can be extended in several other directions. Firstly, the model can be extended to more than a single break, allowing then for the existence of more than two segmented trends in the data. However, for the validity of the type of long-memory (fractional integration) model we use in this application it is necessary that the data span a sufficiently long period of time to detect the dependence across time of the observations; given the sample size of the series employed here, the inclusion of two or more breaks would result in relatively short subsamples, therefore invalidating the analysis based on fractional integration. Secondly, other forms of fractional integration can also be examined. For instance, the detrended series may be  $I(d)$  with a pole or singularity in the spectrum occurring at a frequency away from zero. Non-linear trends can also be considered. Work in these directions is now under progress.



## Endnotes

1. In the context of autocorrelated disturbances, we can use the Prais-Winsten (1954) transformation, in order to obtain a t-statistic, which converges in distribution to a  $N(0,1)$  random variable. However, as noted by Park and Mitchell (1980) and Woodward and Gray (1993), significant size distortions appear in the test statistic when the coefficient is close to 1. Canjels and Watson (1997) have proposed a conservative approach that controls these size distortions.
2. McKittrick (2001) states the importance of analyzing the time series properties of the surface temperatures when testing for trends. He showed that inference based on naïve modelling strategies can lead to unreliable conclusions about the warming effect. See, also Zheng and Basher (1999).
3. An  $I(0)$  process is defined here as a covariance stationary process with spectral density function that is positive and finite at the zero frequency. It thus includes the standard white noise case, but also stationary AR, MA, stationary ARMA, etc.
4. The condition  $u_t = 0, t \leq 0$  is required for the Type II definition of fractional integration. For an alternative definition (Type I) see Marinucci and Robinson (1999).
5. Robinson and Iacome (2005) examine the interaction of long memory with deterministic trends.
6. Diebold and Inoue (2001), Granger and Hyung (2004) and others have shown that fractional integration and structural break models can be easily confused.
7. Note that the lack of long memory behaviour in these two cases may be partly due to the competition between the fractional differencing parameter and the AR coefficient in describing the time dependence.

## References

- Bloomfield, P., 1992: Trends in global temperatures. *Climatic Change*, **21**: 275-287.
- Canjels, E. and M.W. Watson, 1997: Estimating deterministic trends in the presence of serially correlated errors. *Review of Economics and Statistics*, **79**: 184-200.
- Diebold, F.X. and A. Inoue, 2001: Long memory and regime switching. *Journal of Econometrics*, **105**: 131-159.
- Dueker, M.J. and P.K. Asea, 1998: Non-monotonic long memory dynamics in black market premia, Typescript, The Federal Reserve Bank of St. Louis.
- Fomby, T.B. and T.J. Vogelsang, 2002: The application of size-robust trend statistics to global warming temperature series. *Journal of Climate*, **15**: 117-123.
- Ghil, M. and R. Vautard, 1991: Interdecadal oscillations and the warming trend in global temperature time series. *Nature*, **350**: 324-327.
- Gil-Alana, L.A., 2003: An application of fractional integration to a long temperature time series. *International Journal of Climatology*, **23**: 1699-1710.
- Gil-Alana, L.A., 2005: Statistical modelling of the temperatures in the northern hemisphere using fractionally integrated techniques. *Journal of Climate*, **18**: 5357-5369.
- Gil-Alana, L.A., 2007: Fractional integration and structural breaks at unknown periods of time. Forthcoming in *Journal of Time Series Analysis*.
- Granger, Clive W.J. and Z. Ding, 1996: Varieties of long memory models, *Journal of Econometrics*, **73**: 61-77.
- Granger, C.W.J. and Hyung, N., 2004: Occasional structural break and long memory with an application to the S&P 500 absolute stock returns. *Journal of Empirical Finance*, **11**: 399-421.

- Hansen, J. and S. Lebedeff, 1987: Global surface air temperatures. Update through 1987. *Geophysical Letters*, 15: 323-326.
- Hasselmann, K., 1993: Optimal fingerprints for the detection of time dependent climate change. *Journal of Climate*, 6: 1957-1971.
- Hegerl, G.C., H. von Storch, K. Hasselmann, B.D. Santer, U. Cubasch and P.D. Jones, 1996: Detecting greenhouse gas induced climate change with an optimal fingerprint method. *Journal of Climate*, 9: 2281-2306.
- Hegerl, G.C., K. Hasselmann, U. Cubasch, J.F.B. Mitchell, E. Roeckner, R. Voss and J. Waskewitz, 1997: On multi-fingerprint detection and attribution of greenhouse gas and aerosol forced climatic change. *Climate Dynamics*, 13: 613-634.
- Jones, P.D., 1988: Hemispheric surface air temperature variations. Recent trends and an update to 1987. *Journal of Climate*, 1: 654-660.
- Jones, P.D., 1994: Hemispheric surface air temperature variations. A reanalysis and an update to 1993. *Journal of Climate*, 7: 1794-1802.
- Jones, P.D. and K.R. Briffa, 1992: Global surface air temperature variations during the twentieth century, Part I, spatial, temporal and seasonal details. *The Holocene*, 2: 165-179.
- Jones, P.D. and G.C. Hegerl, 1998: Comparisons of two methods of removing anthropogenically related variability from the near surface observational temperature field. *Journal of Geophysical Research*, 103: 777-786.
- Jones, P.D., T.J. Osborn and K.R. Briffa, 1997: Estimating sampling errors in large scale temperature averages. *Journal of Climate*, 10: 2548-2568.

- Jones, P.D., S.C.B. Raper, R.S. Bradley, H.F. Diaz, P.M. Kelly and T.M.L. Wigley, 1986a: Northern hemisphere surface air temperature variations: 1851-1984. *Journal of Climate and Applied Meteorology*, 25: 161-179.
- Jones, P.D., S.C.B. Raper and T.M.L. Wigley, 1986b: Southern hemisphere surface air temperature variations: 1851-1984. *Journal of Climate and Applied Meteorology*, 25: 1213-1230.
- Jones, P.D., T.M.L. Wigley and P.B. Wright, 1986c: Global temperature variations between 1861 and 1984. *Nature*, 322: 430-434.
- Jones, P.D., D.E. Parker, T.J. Osborn and K.R. Briffa, 2006: Global and hemispheric temperature anomalies-land and marine instrumental records. In Trends. A Compendium of Data on Global Change. Carbon Dioxide Information Analysis Center, Oak Ridge National Laboratory, U.S. Department of Energy, Oak Ridge, Tenn, U.S.A.
- Kaufmann, R.K., H. Kauppi and J.H. Stock, 2005: The relation between radiative forcing and temperature: what do statistical analyses of the observational record measure? *Climatic Change*, in review.
- Koscielny-Bunde, E., A. Bunde, S. Havlin, H.E. Roman, Y. Goldreich and H.J. Schellnhuber, 1998: Indication of a universal persistence law governing atmospheric variability, *Physics Review Letters*, 81: 729-732.
- Lewis, P.A.W. and B.K. Ray, 1997: Modelling long-range dependence, nonlinearity and periodic phenomena in sea surface temperatures using TSMARS. *Journal of the American Statistical Association*, 92: 881-893.
- Maraun, D., H.W. Rust and H. Timmer, 2004: Tempting long memory on the interpretation of DFA results. *Nonlinear Processes in Geophysics*, 11: 495-503.

- Marinucci, D. and P.M. Robinson, 1999: Alternative forms of fractional Brownian motion. *Journal of Statistical Planning and Inference*, 80: 111-122.
- McKittrick, R., 2001: Inference about trends in temperature data after controlling for serial correlation and heteroscedasticity variance. *Proceedings of the Russian Geographical Society*, 163: 16-24.
- Nicholls, N., G.V. Gruza, J. Jouzel, T.R. Karl, L.A. Ogallo and D.E. Parker, 1996: Observed climate variability and change, in *Climate Change 1995: The Science of Climate Change*, edited by J.T. Houghton, L.G.M. Filho, B.A. Callander, N. Harris, A. North, G.R. and K.-Y. Kim, 1995: Detection of forced climate signals, Part II., Simulation results. *Journal of Climate*, 6: 409-417.
- North, G.R., K.-Y. Kim, S. P. Shen and J.W. Hardin, 1995: Detection of forced climate signals, Part I: filter theory. *Journal of Climate*, 6: 401-408.
- Park, R.E. and B.M. Mitchell, 1980: Estimating the autocorrelated error model with trended data. *Journal of Econometrics*, 13: 185-201.
- Pelletier, J. and D. Turcotte, 1999: Self-affine time series II. Applications and Models. *Advances in Geophysics*, 40: 91-166.
- Percival, D.B., J.E. Overland and H.O. Mofjeld, 2004: Modelling North Pacific climate time series, to appear in *Time Series Analysis and Applications to Geophysical Systems*, edited by D.R. Brillinger, E.A. Robinson and F.P. Schoenberg, Springer-Verlag.
- Pethkar, J.S. and A.M. Selvam, 1997: Nonlinear dynamics and chaos. Applications for prediction of weather and climate, Proc. TROPMET 97, Bangalore, India.
- Prais, S.J. and C.B. Winsten, 1954: Trend estimators and serial correlation, Cowles Commission Monograph, No. 23, New Haven CT, Yale University Press.

- Robinson, P. and F. Iacome, 2005: Cointegration in fractional systems with deterministic trends. *Journal of Econometrics*, 129: 263-298.
- Rust, B.W., 2003: Separating signal from noise in global warming. *Computing Science and Statistics*, 35: 263-277.
- Santer, B.D., K.E. Taylor, T.M.L Wigley, J.E. Penner, P.D. Jones and U. Cubasch, 1995: Towards the detection and attribution of an anthropogenic effect on climate. *Climate Dynamics*, 12: 79-100.
- Schlesinger , M.E. and N. Ramankutty, 1994: Low-frequency oscillation in the global climate system of period 65-70 years. *Nature*, 367: 723-727.
- Smith, R.L., 1993: Long-range dependence and global warming. Statistics for the Environment (ed. Barnett, V. And Turkman, K.F., pub. J. Wiley), 141-161.
- Smith, R.L. and F.-L. Chen, 1996: Regression in long-memory time series. In Athens Conference on Applied Probability and Time Series, Volume II: Time Series Analysis in Memory of E.J. Hannan, edited by P.M. Robinson and M. Rosenblatt, Springer *Lecture Notes in Statistics*, 115, 378-391.
- Stern, D.I. and R.K. Kaufmann, 2000: Detecting a global warming signal in hemispheric temperature series. A structural time series analysis. *Climatic Change*, 47: 411-438.
- Woodward, W.A., Gray, H.L. 1993: Global warming and the problem of testing for trend in time series data. *Journal of Climate*, 6: 953-962.
- Wu, W.B. and Z. Zhao, 2007: Inference of trends in time series. *Journal of the Royal Statistical Society, Series B*, 69: 391- 410.
- Zheng, X. and R. E Basher, 1999: Structural time series models and trend detection in global and regional temperature series. *Journal of Climate*, 12: 2347-2358.

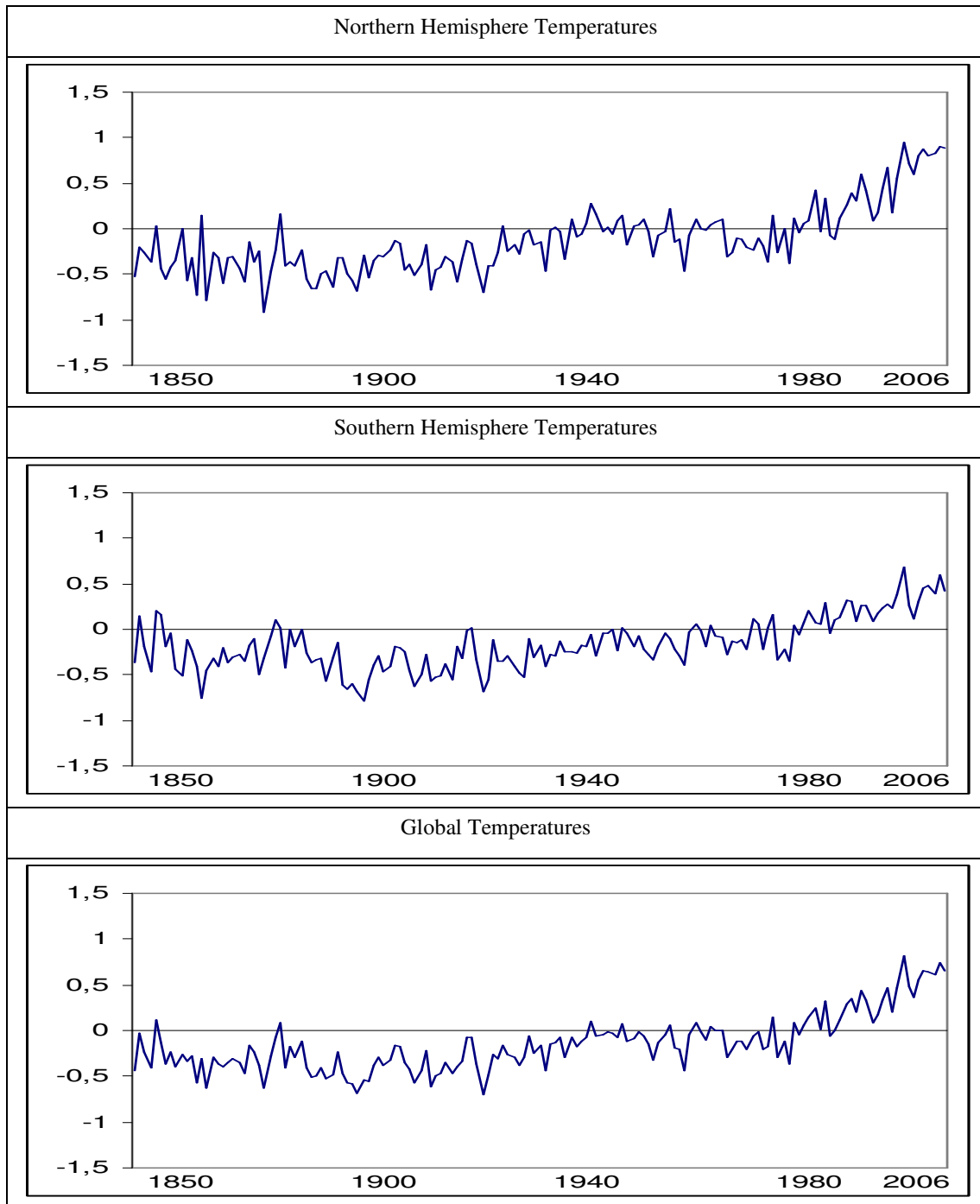
**Figure captions list:**

Figure 1: Original time series. The series are anomalies with respect to 1961-1990.

Figure 2: Original series and linear fitted trends.

Figure 3: Original series and segmented trends in case of white noise disturbances.

Figure 4: Original series and segmented trends in case of AR(1) disturbances.



**Figure 1: Original time series. The series are anomalies with respect to 1961-1990.**



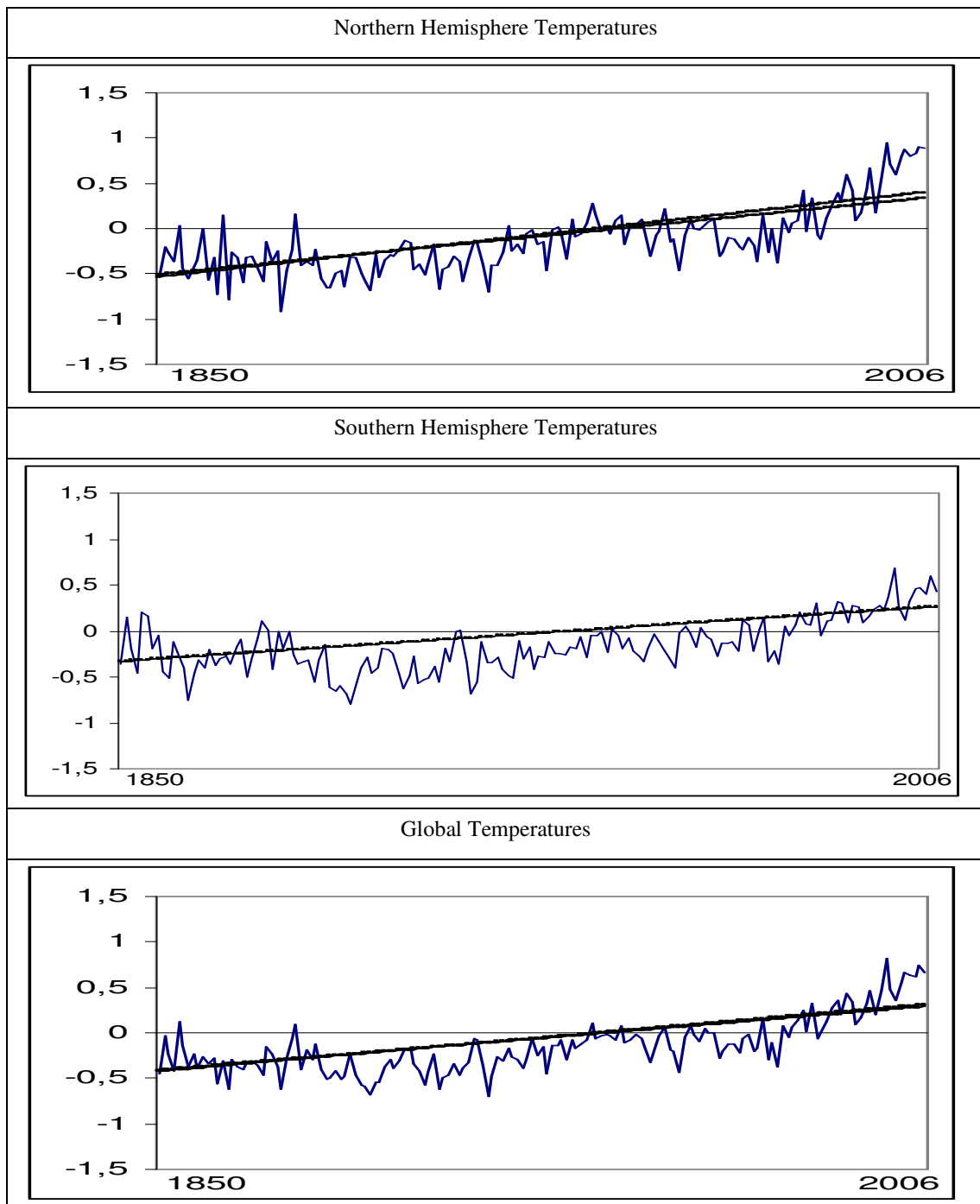


Figure 2: Original series and the fitted linear trends

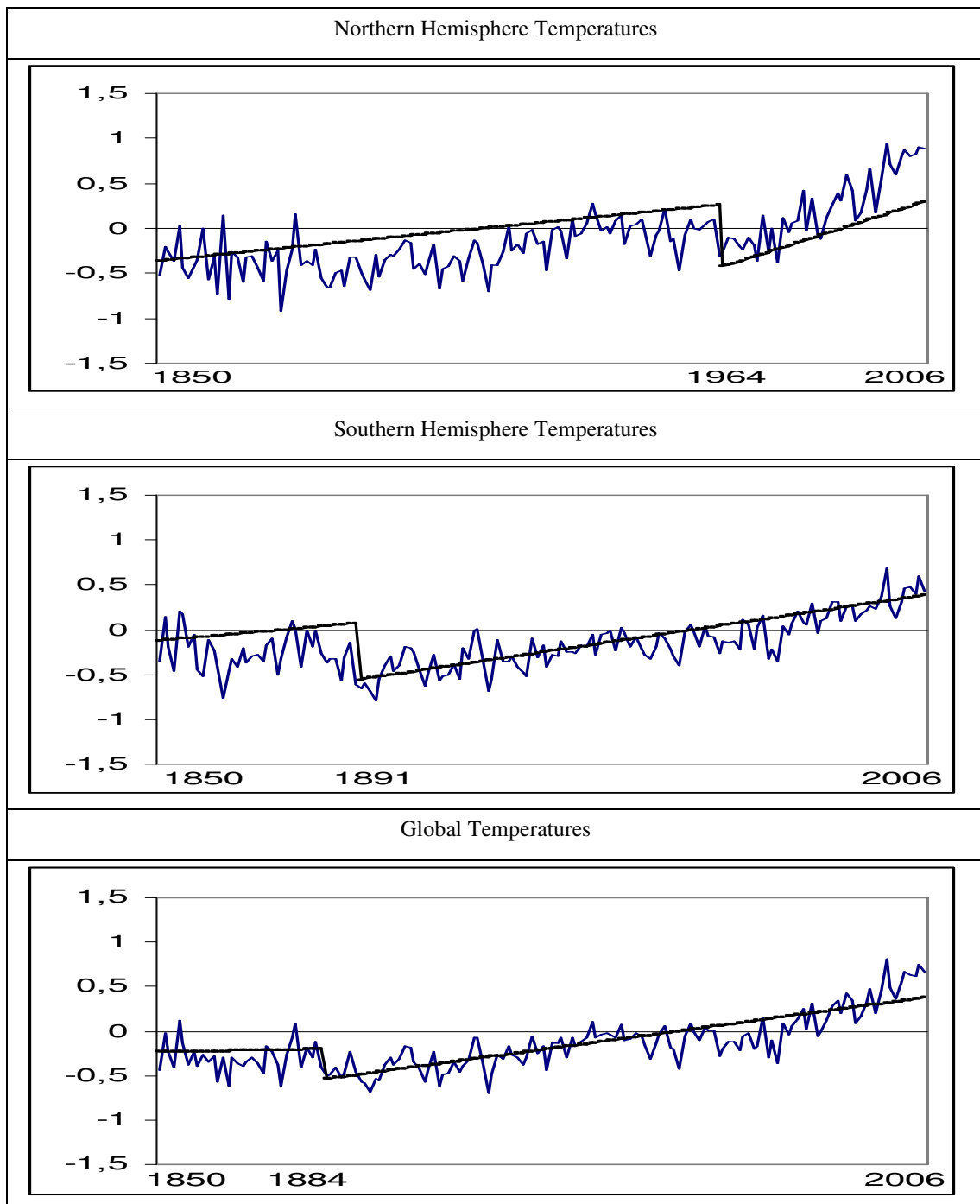


Figure 3: Original series and segmented trends in case of white noise disturbances

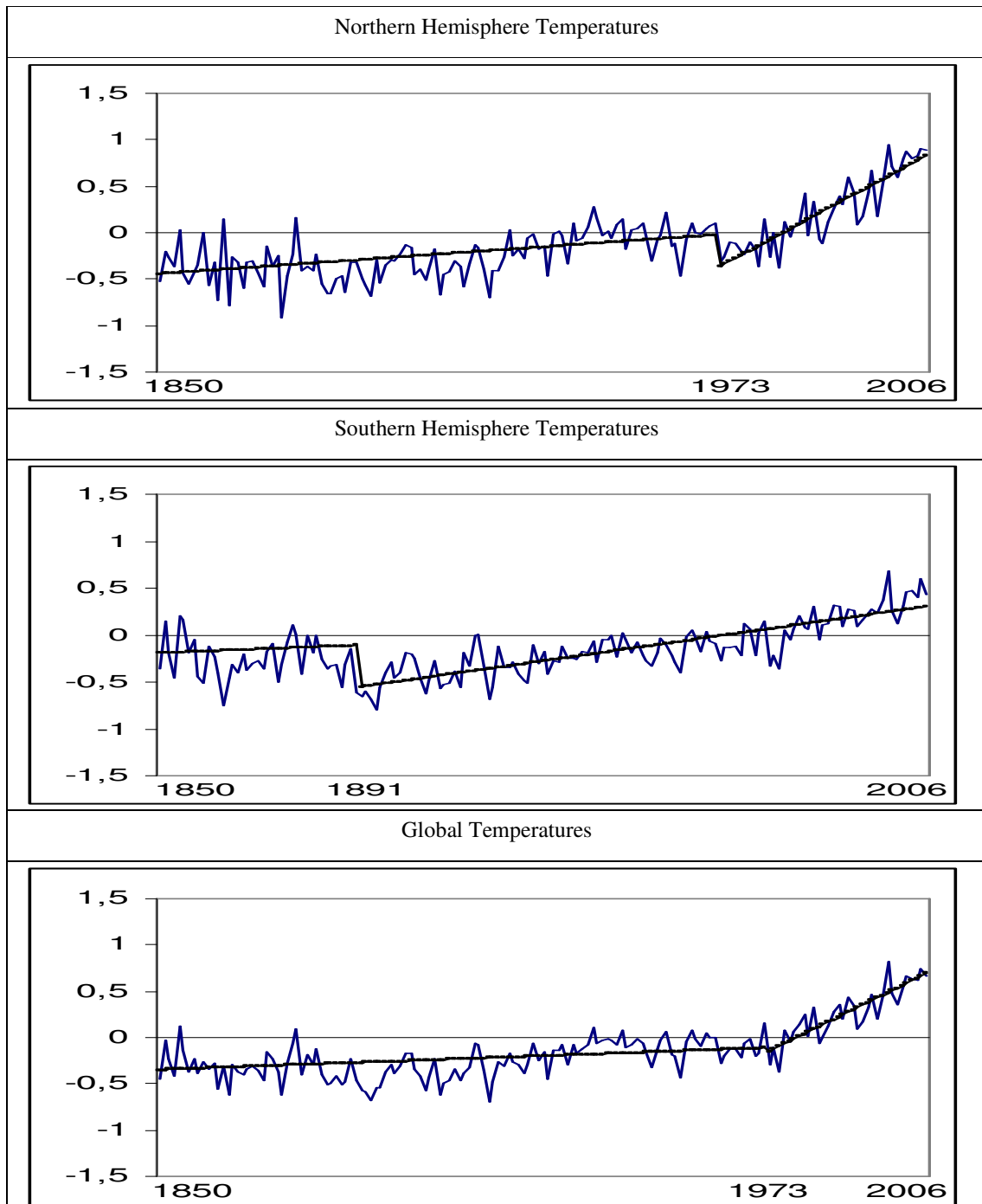


Figure 4: Original series and segmented trends in case of AR(1) disturbances

**Table 1: Estimates of  $d$  and (95 %-) confidence bands for the three series. In bold, the estimated values of  $d$  based on the Whittle function.**

i) White noise disturbances			
	No det. Terms	An intercept	A linear trend
Northern H.	[0.397 ( <b>0.449</b> ) 0.517]	[0.381 ( <b>0.430</b> ) 0.495]	[0.299 ( <b>0.367</b> ) 0.455]
Southern H.	[0.416 ( <b>0.480</b> ) 0.569]	[0.405 ( <b>0.466</b> ) 0.556]	[0.338 ( <b>0.418</b> ) 0.529]
Global T.	[0.459 ( <b>0.516</b> ) 0.593]	[0.442 ( <b>0.497</b> ) 0.575]	[0.379 ( <b>0.452</b> ) 0.549]
ii) AR(1) disturbances			
	No det. Terms	An intercept	A linear trend
Northern H.	[0.498 ( <b>0.580</b> ) 0.685]	[0.480 ( <b>0.559</b> ) 0.668]	[0.427 ( <b>0.526</b> ) 0.653]
Southern H.	[0.408 ( <b>0.493</b> ) 0.595]	[0.397 ( <b>0.474</b> ) 0.572]	[0.299 ( <b>0.403</b> ) 0.530]
Global T.	[0.497 ( <b>0.582</b> ) 0.688]	[0.481 ( <b>0.560</b> ) 0.664]	[0.413 ( <b>0.516</b> ) 0.641]

**Table 2: Estimates of the coefficients associated to the time trend for each series. t-values in parenthesis. In bold, significant coefficients at the 5% level.**

i) White noise disturbances				
	Intercept	Time Trend	Estimated d	AR coefficient
Nothern H.	<b>-0.54327</b> (-4.860)	<b>0.00600</b> (4.853)	0.367	-----
Southern H.	<b>-0.33328</b> (-3.155)	<b>0.00380</b> (3.106)	0.418	-----
Global T.	<b>-0.42778</b> (-4.138)	<b>0.00470</b> (3.767)	0.452	-----
ii) AR(1) disturbances				
	Intercept	Time Trend	Estimated d	AR coefficient
Nothern H.	<b>-0.51509</b> (-3.200)	<b>0.00541</b> (2.455)	0.526	-0.276
Southern H.	<b>-0.33628</b> (-3.296)	<b>0.00384</b> (3.301)	0.403	0.025
Global T.	<b>-0.41471</b> (-3.529)	<b>0.00448</b> (2.840)	0.516	-0.111

**Table 3: Estimates based on fractional integration with a structural break. t-values in parenthesis. In bold, significant coefficients at the 5% level.**

i) White noise disturbances									
		First sub-sample				Second sub-sample			
	Break	Int.	L.Trend	d	AR	Int.	L.Trend	d	AR cf.
North	1964	<b>-0.36874</b> (-3.357)	<b>0.00545</b> (3.316)	0.348	-----	<b>-2.45729</b> (-2.719)	<b>0.01750</b> (2.495)	0.479	-----
South	1891	-0.12881 (-0.793)	0.00473 (0.665)	0.437	-----	<b>-0.90066</b> (-8.290)	<b>0.00816</b> (7.631)	0.322	-----
Global	1884	-0.24101 (-1.583)	0.00107 (1.308)	0.469	-----	<b>-0.80591</b> (-7.004)	<b>0.00750</b> (6.178)	0.392	-----
ii) AR(1) disturbances									
		First sub-sample				Second sub-sample			
	Break	Int.	L.Trend	d	AR	Int.	L.Trend	d	AR cf.
North	1963	<b>-0.45158</b> (-5.750)	<b>0.00373</b> (3.353)	0.257	-0.147	<b>-3.64315</b> (-11.63)	<b>0.02848</b> (12.430)	0.032	-0.011
South	1891	<b>-0.19745</b> (-2.253)	-0.00208 (-0.572)	0.000	0.214	<b>-0.86801</b> (-11.36)	<b>0.00744</b> (10.403)	0.072	0.277
Global	1973	<b>-0.35264</b> (-4.445)	<b>0.00191</b> (1.875)	0.283	0.075	<b>-3.42123</b> (-10.44)	<b>0.02619</b> (11.294)	0.000	-0.087

**Table 4: Warming effect coefficients according to the different models. In bold, significant coefficients at the 5% level.**

	d = 0		Fractional d		Fractional d + Segmented trends			
					1 <sup>st</sup> sub-sample		2 <sup>nd</sup> sub-sample	
	W.N.	AR(1)	W.N.	AR(1)	W.N	AR(1)	W.N.	AR(1)
NORTH	<b>0.60°C</b>	<b>0.69°C</b>	<b>0.60°C</b>	<b>0.54°C</b>	<b>0.54°C</b>	<b>0.37°C</b>	<b>1.75°C</b>	<b>2.84°C</b>
SOUTH	<b>0.39°C</b>	<b>0.70°C</b>	<b>0.38°C</b>	<b>0.38°C</b>	0.47°C	-0.20°C	<b>0.81°C</b>	<b>0.74°C</b>
GLOBAL	<b>0.48°C</b>	<b>0.69°C</b>	<b>0.47°C</b>	<b>0.44°C</b>	0.10°C	<b>0.19°C</b>	<b>0.75°C</b>	<b>2.61°C</b>