WCTE
WORLD CONFERENCE ON TIMBER ENGINEERING
2010

# EVALUATION OF FAILURE CRITERIA IN WOOD MEMBERS

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**ABSTRACT:** Multi-axial stress conditions exist in almost all engineering applications, but design procedures, in most cases, assume uniaxial stress conditions. This assumption is not adequate for orthotropic materials such as timber. Initial failure mode in lumber is usually tension perpendicular to the grain or axial tension around natural defects such as knots. Strength properties of lumber have been obtained from clear-wood specimens and in-grade testing of full-scale lumber whereas failure criteria are usually those originally developed for composite materials. Some of the existing failure criteria are relatively easy to use, but prove valid only under special orthotropic conditions. Some studies assume that the tension and compression strengths to be equal, which is not the case for wood. Other criteria are usually based on quadric surfaces, in which certain constraints are taken strictly from geometrical considerations to achieve a closed failure envelope. Since these conditions are not defined from a physical stand point, some parts of the resulting curve leads to errors in failure predictions. These quadric criteria also require evaluation of interaction coefficients for bi-axial stress conditions. Evaluation of these interaction coefficients demands extensive experimental testing, which limits their practical application. Some proposals have been made to deal with the interaction approach either by evaluating the interaction factor in terms of uniaxial strength or by proposing a given value in the absence of experimental data. The proposed paper reviews existing failure criteria. Their relevance to timber structural members is discussed and validated against experimental data. Statistical analyses are performed to assess their adequacy.

**KEYWORDS:** Failure criteria, Failure models, Wood members, Stress interaction, Biaxial loading

## 1 INTRODUCTION

Multiaxial stress states exist in engineering applications, but most design procedures are based on uniaxial stress states. This approach works well when one stress is much larger than the other. But when combined loading is involved, multiaxial stress states should be considered.

Failure criteria or models for isotropic materials that exist in the literature provide relatively reliable predictions of failure (such as von Mises). The non-isotropic models are mainly for composite materials, and may be applied to wood – an orthotropic material. There are few models that were developed specifically for wood (Norris, 1950; van der Put, 2005).

Phenomenological strength criteria apply to phenomenon of failure and do not explain the mechanism of failure. They are mathematically represented by an expression describing a strength surface. The expression contains normalized values which correspond to a closed failure envelope of a uniaxially-loaded material strength. The main advantage of the criteria is its simplicity. The most widely used failure criterion is of quadratic form. For a uniaxially-loaded material, the following allowable strength values are required: longitudinal compression ($X_c$ ) and tension ($X_t$), transversal compression ($Y_c$ ) and tension ($Y_t$ ), and shear (S). In this paper, strength values in the radial and tangential directions are assumed to be the same.

Hinton et al. (2004) extensively reviewed the existing criteria for composite failure. The work resulted in what is being called "*The World Wide Failure Exercise*". In this study, we validated existing failure models against experimental results and evaluated their application to wood members.
Objectives:
The specific objectives were:
(1) to evaluate existing failure models to wood applications, and
(2) to validate the models using experimental data.

## 2 PROCEDURE

### 2.1 EXPERIMENTAL PROTOCOL

The literature is rich in data for uniaxial stress states but is weak in multiaxial stress states of wood. Eberhardsteiner (2002) reported an extensive test data for cruciform specimens of clear spruce wood in biaxial loading (Fig. 1). He conducted 450 individual tests. The

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tests cover a wide range of stress states for an orthotropic material under plane stress. We used Eberhardsteiner (2002) test data to evaluate and compare failure criteria models that are available in the literature.

The specimens were 140x140mm clear wood without any visible imperfections. Because of this restriction, the length of the specimen was inline with the longitudinal-radial plane. The thickness of each specimen was 4.5mm for biaxial tension and combined tensile stress tests but was 7.5- 9.5mm for the compression stress tests. The angle between the direction of loading and the direction of grain of the wood was varied for different tests.

## 2.2 STRENGTH VALUES

Failure models were evaluated using lowest 5% exclusion limit uniaxial strength values (Table 1). This values were for clear spruce wood, and are obtained from the experiments by Eberhardsteiner (2002). The properties were uniaxial tests in the x and y directions, and the shear strength values (S) were uniaxial tests at 45º.

***Table 1.*** *Experimental material strength values from Eberhardsteiner (2002) uniaxial tests.*

| Wood property | Mean ($N/mm^2$) | Lowest 5% Exclusion Limit ($N/mm^2$) |
|---|---|---|
| $X_c$ | 41.67 | 36.50 |
| $Y_c$ | 7.00 | 5.13 |
| $X_t$ | 62.33 | 41.60 |
| $Y_t$ | 4.70 | 3.96 |
| S | 8.08 | 7.28 |

## 2.3 TRANSFORMATION OF STRENGTH VALUES TO THE COORDINATES OF THE MATERIAL

Test strength data that were off-axis ($\sigma_1, \sigma_2$ not parallel to the grain) were transformed into on-axis values ($\sigma_x, \sigma_y$ and $\tau_{xy}$) using the transformation equation (Bodig and Jayne, 1982) expressed as

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} = \begin{bmatrix} m^2 & n^2 & 2mn \\ n^2 & m^2 & -2mn \\ -mn & mn & m^2-n^2 \end{bmatrix} \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_6 \end{bmatrix} \qquad (1)$$

where, $m = \cos\phi$, $n = \sin\phi$, $\sigma_x$ and $\sigma_y$ *are* compression and tension test values along the plane of the member, respectively, and 1,2, etc represent off-axis values for a coordinate system rotated at an angle $\phi$ (Fig.2), $\tau_{xy}$ and $\tau_6$ are shear stresses, on- and off-axis coordinates, respectively.

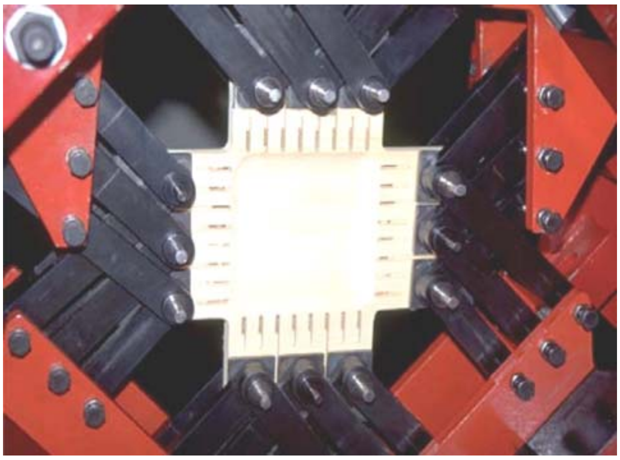

***Figure 1.*** *Experimental set up (taken from Eberhardsteiner, 2002).*

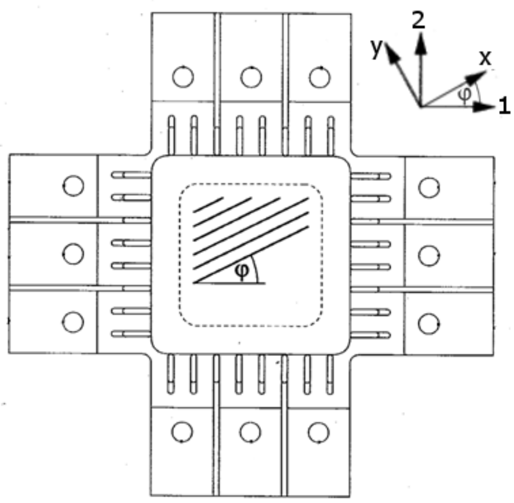


***Figure 2.*** *Material coordinates (x,y) and experiment coordinates (1,2) (from Eberhardsteiner, 2002).*

## 2.4 STRENGTH RATIO

To compare the different failure criteria, a strength ratio, *R*, similar to the inverse of that defined by Tsai & Wu (1992) is defined. For materials loaded within the linearly elastic range, R is the ratio between the *capacity* of the material and the *demand* side of the equation. "*Demand*" refers to the load, which the member is subjected or expected to carry. For the material to remain safe, the *capacity* side should be greater than the *demand* side. Therefore, if R $\leq$ 1.0, the material has not failed but if R>1.0, the material has failed. Since each test in Eberhardsteiner (2002) experiments was carried to failure, their corresponding R values should be greater than one. For example, for given test values of $\sigma_x, \sigma_y$ and $\tau_{xy}$, the capacity can be calculated as $\frac{\sigma_x}{R}, \frac{\sigma_y}{R}$ and $\frac{\tau_{xy}}{R}$, and R<1.0.

The value of R can be solved given either strength test values or material capacities. For example, the equation for a quadratic failure criterion is expressed as

$$\left(\frac{1}{R}\frac{\sigma_x}{X}\right)^2+\left(\frac{1}{R}\frac{\sigma_y}{Y}\right)^2+\left(\frac{1}{R}\frac{\tau_{xy}}{S}\right)^2=1.0 \qquad (2)$$

# 3 FAILURE CRITERIA

For preliminary design purposes, it is good to have reliable models that require minimum input data obtained from simple tests to predict strength values for members subjected to combined normal and shear loading. The anisotropic strength failure models that exist in the literature are extensions of isotropic yield models. The octahedral shear stress yield theory proposed by Von Mises (1928) has significantly influenced the theory of failure of composite materials (Rowlands, 1985). This section presents some of these theories that apply to wood strength and their validations will be presented later.

## 3.1 LINEAR MODELS

The simplest model is the linear interaction equation for biaxial stresses (Aicher and Klöck, 2001), and is expressed as

$$\frac{\sigma_x}{X}+\frac{\sigma_y}{Y}+\frac{\tau_{xy}}{S}=1.0 \qquad (3)$$

where, the numerators are on-axis test values and the denominators are allowable strength values.

## 3.2 QUADRATIC MODELS

Most failure criteria models are quadratic polynomials. The simplest one is of the form of an ellipsoidal envelope expressed as (Aicher and Klöck, 2001)

$$\left(\frac{\sigma_x}{X}\right)^2+\left(\frac{\sigma_y}{Y}\right)^2+\left(\frac{\tau_{xy}}{S}\right)^2=1.0 \qquad (4)$$

The above failure criterion represents a special case of a more general quadratic form. Equation (4) is for a case when the tension and compression strength values are the same, and neglects the interaction between the normal stresses, $\sigma_x$ and $\sigma_y$.

### 3.2.1 Tsai-Hill Model

Hill (1950) reported a theory of failure for plastic anisotropy, based on Von Mises-Hencky's distortion energy theory (Von Mises, 1928). His assumption was that a crystal metallic structure that is initially isotropic becomes aligned and eventually behaves as an anisotropic. Hill (1950) assumed that the yield stresses were the same in tension and compression. His formulation contained interaction between stresses, and therefore involved combined failure modes. Azzi and Tsai (1965) later adapted Hill's (1950) theory for composites. The Tsai-Hill (1965) failure model is expressed as

$$\left(\frac{\sigma_x}{X}\right)^2-\frac{\sigma_x\sigma_y}{X^2}+\left(\frac{\sigma_y}{Y}\right)^2+\left(\frac{\tau_{xy}}{S}\right)^2=1.0 \qquad (5)$$

Equation (5) includes uniaxial strength parallel to the grain (X) and perpendicular to the grain (Y). The above model is based on the model proposed by Hill (1950), which is expressed as

$$\left(\frac{\sigma_x}{X}\right)^2-\sigma_x\sigma_y\left(\frac{1}{X^2}+\frac{1}{Y^2}-\frac{1}{Z^2}\right)+\left(\frac{\sigma_y}{Y}\right)^2+\left(\frac{\tau_{xy}}{S}\right)^2=1 \qquad (2)$$

For composite materials, the Y and Z strength values (axes perpendicular to the longitudinal direction) are considered to be the same. This simplification would be adequate for wood because radial and tangential directions are usually assumed to be the same. Given this assumption, therefore, only Tsai-Hill (1965) theory is discussed in this paper.

Tsai-Hill (1965) theory assumes that the yield stresses are the same in tension and compression, which is not the case in wood. Using the procedure outlined by Rowlands (1985), the corresponding tensile or compressive strengths are taken for each quadrant in the $\sigma_x$ and $\sigma_y$ stress space, and results in four different equations (one for each quadrant).

Unlike the linear and quadratic models, this approach includes interaction of stresses. The interaction is, however, fixed. Other models, discussed below, include independent interactions among all stress components.

### 3.2.2 Norris Model

Norris (1950) postulated that failure in wood would occur under plane stress if any of the following equations is satisfied:

$$\left(\frac{\sigma_x}{X}\right)^2-\frac{\sigma_x\sigma_y}{XY}+\left(\frac{\sigma_y}{Y}\right)^2+\left(\frac{\tau_{xy}}{S}\right)^2=1$$

$$\left(\frac{\sigma_x}{X}\right)^2=1 \quad \text{or} \quad \left(\frac{\sigma_y}{Y}\right)^2=1 \qquad (3)$$

As with most orthotropic strength criteria, the first quadratic equation in Eq. (7) is similar in form to Eq. (5). Van der Put (2005) indicated that equations by Norris (Eq.7), although extensively used, are not generally valid for wood.

### 3.2.3 Tsai-Wu Model

Tsai and Wu (Tsai, 1992) proposed a failure criterion that included additional stress terms. The criterion assumed a failure surface of the form

$$f(\sigma)=F_i\sigma_i+F_{ij}\sigma_i\sigma_j \qquad i,j=x,y,z \quad (4)$$

Where, $F_i$ and $F_{ij}$ are second and fourth order strength tensors. The linear stress terms account for differences in tensile and compressive strengths. The quadratic stress terms are similar to those in Tsai-Hill (1965) model (Eq. 6), which defines an elliptical stress space. Under plane-stress conditions, this failure criterion becomes

$$\left(\frac{1}{X_t}-\frac{1}{X_c}\right)\sigma_x+\left(\frac{1}{Y_t}+\frac{1}{Y_c}\right)\sigma_y+\frac{1}{X_tX_c}\sigma^2{}_x$$
$$+\frac{1}{Y_tY_c}\sigma^2{}_y+2a_{xy}\sqrt{\left(\frac{1}{X_tX_c}\frac{1}{Y_tY_c}\sigma_x\sigma_y\right)}+\frac{1}{S^2}\tau^2{}_{xy}=1.0 \qquad (9)$$

To determine the interaction coefficient, $a_{xy}$, experimental tests are required. Also, the stability condition, $-1.0 \le a_{xy} \le 1.0$, has to be met in order to obtain a closed envelope.

In the previous failure models (linear or quadratic), the interaction of stress was not included and the failure criteria was defined by simultaneous equations (like in Norris, 1950). In the case of Norris (1950) and Tsai-Hill (1965) models, the stress interaction was fixed. In the case of the Tsai-Wu model (Eq. 9), the interaction coefficient, $a_{xy}$, needs to be defined. The interaction coefficient is difficult to determine. Tsai (1991) proposed $a_{xy}=-0.5$, which corresponds to the generalized Von Misses criterion. In this paper, the Tsai-Wu model was evaluated using $a_{xy}=-0.5$. We also considered $a_{xy}=0.04$ based on Eberhardsteiner (2002) second-order fitting curve obtained for clear spruce wood under biaxial loading.

### 3.2.4 Von Mises Model

Tsai (1988) and Kim (1995) suggested the use of a model, which is similar to Norris (1950) model (Eq.(7). The only difference is that the shear term is multiplied by 3. The model is expressed as

$$\left(\frac{\sigma_x}{X}\right)^2-\frac{\sigma_x\sigma_y}{XY}+\left(\frac{\sigma_y}{Y}\right)^2+3\left(\frac{\tau_{xy}}{S}\right)^2=1 \qquad (5)$$

### 3.2.5 Van der Put Model

Van der Put (2005) proposed a general tensor polynomial failure model for wood, and is expressed as

$$\left(\frac{1}{X_t}-\frac{1}{X_c}\right)\sigma_x+\left(\frac{1}{Y_t}-\frac{1}{Y_t}\right)\sigma_y+\frac{1}{X_tX_c}\sigma_x{}^2+\frac{1}{Y_tY_c}\sigma_y{}^2+\frac{1}{S^2}\tau_{xy}{}^2=1 \qquad (11)$$

Equation (11) is similar to Eq. (9) proposed by Tsai-Wu (1992). Both models are based on the same tensor theory. The only difference between the two models is that the interaction coefficient, $a_{xy}$, is equal to zero in Eq. (11), as proposed by Van der Put (2005).

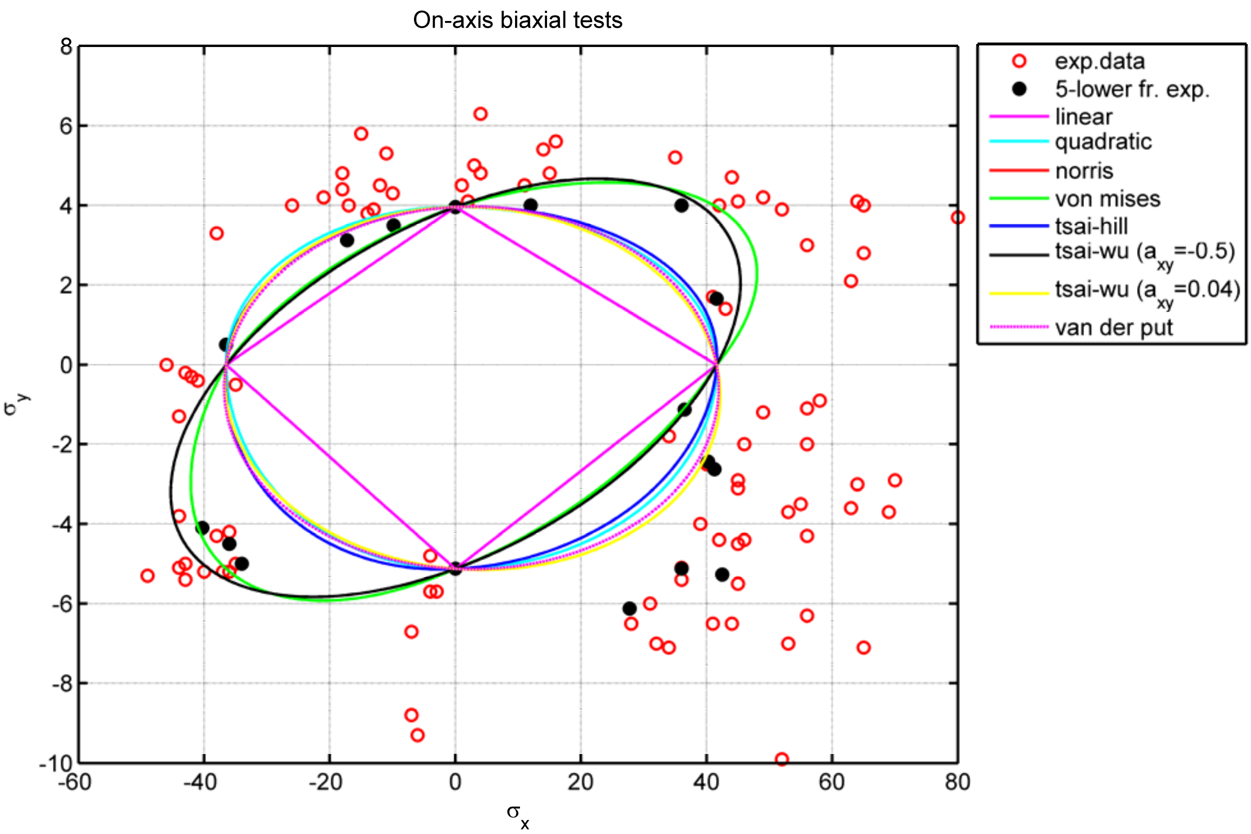


***Figure 3.*** *Plot of the failure criteria with the on-axis experimental data from Eberhardsteiner (2002).*

# 4 RESULTS AND DISCUSIONS

Failure envelopes for the different models presented in Sect. 3, experimental strength values obtained from Eberhardsteiner (2002), and the lowest 5% exclusion limit values are shown in Figure 3. The failure envelopes are based on the lowest 5% exclusion limit of uniaxial strengths values. The strength values are expressed in the material coordinates, x-axis corresponds to the longitudinal axis (parallel to the grain) of the wood, and y-axis refers to perpendicular to the grain.

The failure envelopes consist of linear and quadratic formations. Most of the quadratic failure envelopes are generally similar with the exception of the models by Von Mises (1928) and Tsai-Wu (1992), which includes an interaction coefficient of -0.5. These two models are more elliptical than the others. The predictions from these two models fit the data reasonably well in the first and third quadrants but the lowest 5% exclusion limit values are located mostly inside the envelope, which means the model predicts higher strength than the actual capacity of the material.
The predictions of the other models are smaller than the lowest 5% exclusion limit values, which means the model predicts lower than the actual capacity of the material especially in the fourth quadrant.

## 4.1 COMPARISON OF THE CRITERIA BY MEANS OF A STRENGTH RATIO

The models are further compared using the previously defined (in Sect. 2.4) strength ratio, R. Values of R>1 represent failure.
The failure criteria of each model are compared against the following test data obtained from Eberhardsteiner (2002):

(1) biaxial test data, all orientations (W Series),
(2) test data on-axis loading only (O Series).

The mean and standard deviation of the calculated strength ratios of each model are given in Tables 4.1-4.2. The results show that most of the predictions of the models are higher than one with the exception of the linear model, which predicts lower ratios (Table 4.1).
Since the analysed stress states correspond to tests that actually failed, the mean values greater than unity (failure) are expected. The model that predicts well is therefore the one which has an R value closer or equal to 1.0.
A statistical F-test was performed to determine if the predicted mean values were statistically different from the test data. The models whose predictions are statistically difference at 0.05 level are marked with (*) or those that are statistically different at 0.01 are marked with (**) in Tables 4.1 and4.2.

**Table 1.** Means and standard deviations of R values for each model (W Series). Values marked exhibit either significant (*) or highly significant (**) difference.

| Failure criteria | Global | 1st quad. | 2nd quad | 3rd quad | 4th quad |
|---|---|---|---|---|---|
| **Linear** | 1.364 (±0.77) * | 1.431 (±0.61) ** | 0.774 (±0.54) * | 0.942 (±0.6)* | 2.121 (±0.61)** |
| **Quadratic** | 1.192 (±0.32) | 1.139 (±0.3)* | 1.067 (±0.19) | 1.126 (±0.3) | 1.422 (±0.34) |
| **Norris** | 1.174 (±0.38) | 1.02 (±0.23) | 1.176 (±0.22) | 1.009 (±0.25) | 1.597 (±0.43)* |
| **Von Mises** | 1.322 (±0.55) * | 0.869 (±0.31) * | 1.4 (±0.26) * | 1.345 (±0.43)* | 1.911 (±0.49)** |
| **Ttsai-Hill** | 1.191 (±0.33) | 1.129 (±0.29) | 1.08 (±0.19) | 1.111 (±0.29) | 1.445 (±0.35) |
| **Tsai-Wu ($a_{xy}$=-0.5)** | 1.166 (±0.37) | 1.026 (±0.23) | 1.196 (±0.22) | 0.991 (±0.25) | 1.565 (±0.42)* |
| **Tsai-Wu ($a_{xy}$=0.04)** | 1.182 (±0.31) | 1.152 (±0.31)* | 1.09 (±0.19) | 1.119 (±0.3) | 1.353 (±0.32) |
| **van der Put** | 1.181 (±0.31) | 1.144 (±0.3)* | 1.098 (±0.19) | 1.111 (±0.3) | 1.37 (±0.33) |

**Table 2.** Means and standard deviations of R values for each model (O Series). Values marked exhibit either significant (*) or highly significant (**) difference.

| Failure criteria | Global | 1st quad. | 2nd quad | 3rd quad | 4th quad |
|---|---|---|---|---|---|
| **Linear** | 1.841 (±0.45)** | 1.857 (±0.5)* | 1.562 (±0.2)* | 1.711 (±0.44)** | 2.02 (±0.43)** |
| **Quadratic** | 1.414 (±0.27) | 1.447 (±0.29) | 1.227 (±0.14) | 1.354 (±0.24)* | 1.501 (±0.29) |
| **Norris** | 1.414 (±0.39) | 1.176 (±0.2)* | 1.405 (±0.16) | 1.113 (±0.21) | 1.78 (±0.36)* |
| **Von Mises** | 1.414 (±0.39) | 1.176 (±0.2)* | 1.405 (±0.16) | 1.113 (±0.21) | 1.78 (±0.36)* |
| **Ttsai-Hill** | 1.418 (±0.28) | 1.424 (±0.28) | 1.248 (±0.14) | 1.324 (±0.23) | 1.539 (±0.3) |
| **Tsai-Wu ($a_{xy}$=-0.5)** | 1.407 (±0.39) | 1.2 (±0.21) | 1.419 (±0.17)* | 1.088 (±0.22) | 1.755 (±0.35)* |
| **Tsai-Wu ($a_{xy}$=0.04)** | 1.394 (±0.27) | 1.476 (±0.31) | 1.241 (±0.14) | 1.355 (±0.24)* | 1.421 (±0.27) |
| **van der Put** | 1.398 (±0.27) | 1.458 (±0.3) | 1.255 (±0.14) | 1.337 (±0.24)* | 1.448 (±0.28) |

#### 4.1.1. W Series

The W series are the biaxial off-axis and on-axis tests performed by Eberhardsteiner (2002). Table 4.1 shows the mean R ratios for each of the models considered in this study and accounts for the whole stress space, and for each quadrant.

The linear model seems to be the worst predictor in the global space (higher mean values). This condition was also apparent in Fig.3. There is no statistical difference between the linear model and the von Mises' model. Both are the worst models in the global space.

The predictions of the quadratic models are not statistically different when compared in their global stress space. Looking at the predictions in the quadrants: in the first quadrant ($\sigma_x$ and $\sigma_y$, are tension), the Norris model predicted better than the others closely followed by the Tsai-Wu with an interaction factor of -0.5. This is also evident in Fig. 3. The predictions of these two models are statistically significant difference from the predictions of the other models as shown in Table 4.1 The linear model is clearly the worst predictor in this quadrant.

In the second quadrant ($\sigma_x$ is compression and $\sigma_y$ tension), the linear and von Mises models are the worst predictors. The difference of the predictions of the other models is not however statistically different. In the third quadrant (compression in both axes), the linear and von Mises models are the worst models whereas the Tsai-Wu (with -0.5 interaction factor) and Norris models predict well compared to the others.

Due to the high dispersion of results In the fourth quadrant ($\sigma_x$ is tension and $\sigma_y$ is compression), the models did not predict well because the test data in this quadrant were highly dispersed as shown in Fig. 3. The R values are higher than the other quadrants. In this quadrant, the Tsai-Wu with 0.04 interaction coefficient- and van der Put models predict reasonably well.

#### 4.1.2. O Series

This series includes only on-axis biaxial tests, in which the load was applied parallel to the direction of the grain.

In the global stress space, the linear model is the worst predictor. There is no difference in the prediction of the other models. In the first quadrant (on-axis tension), the linear model is the worst and Norris and von Mises models are the best predictors, with a mean R value close to unity.

The quadratic obtains the best mean In the second quadrant (compression in the grain direction, tension perpendicular to the grain) is the best predictor. As is shown in Fig.3, the models with a more elliptical shape predicts well in the third quadrant (on-axis compression). Tsai-Wu with -0.5 interaction factor, von Mises and Norris are the better than others. In the fourth quadrant (parallel tension parallel to the grain and compression perpendicular to the grain), the result are similar to those in the second quadrant. Tsai-Wu with an interaction factor of 0.04, and van der Put are the best predictors.

## 5 CONCLUSIONS

Existing phenomenological failure models for orthotropic materials were applied to clear wood. The predicted values were validated against biaxial test data available in the literature (Eberhardsteiner, 2002).

Statistical analyses were carried out to determine if the difference in their predictions are statistically significant. The linear model is the worst predictor in biaxial loading conditions.

When compared in the global stress space, no significant difference was obtained among the quadratic models but there were significant differences when comparison was made in the quadrants of the $\sigma_x$- $\sigma_y$ stress spac, the following conclusions can be drawn:

- Tsai-Wu (with an interaction factor of -0.5) and Norris were the best models in the first (biaxial tension) and third (biaxial compression) quadrants.
- In the second and fourth quadrants (in which one of the directions is in compression and the other in tension), the models with a close to null interaction factor (van der Put and Tsai-Wu with 0.04 interaction factor) are the best predictors.

The different performance of the different criteria in each zone of the stress space should be taken into account. Further work has to be accomplished in order to provide the designer with valid and adequate failure criteria for wood.

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